CTAO SCHOOL

Gamma-ray production mechanisms

Giada Peron — CTAO School (La Palma) — 17th May 2026

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Gamma-ray production mechanisms

Lectures based on:

G.B. Rybicki & A. P. Lightman “Radiative Processes in Astrophysics” 2004

F. Aharonian “Very High Energy Cosmic Gamma Radiation” 2004

G. Ghisellini “Radiative processes in high-energy astrophysics” 2012

M. Longair “High Energy Astrophysics” 1992

S.R. Kelner Phys. Rev. D 74-3 2006

F. Stecker “Cosmic gamma rays” NASA Special Publication 1977

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What produces gamma rays?

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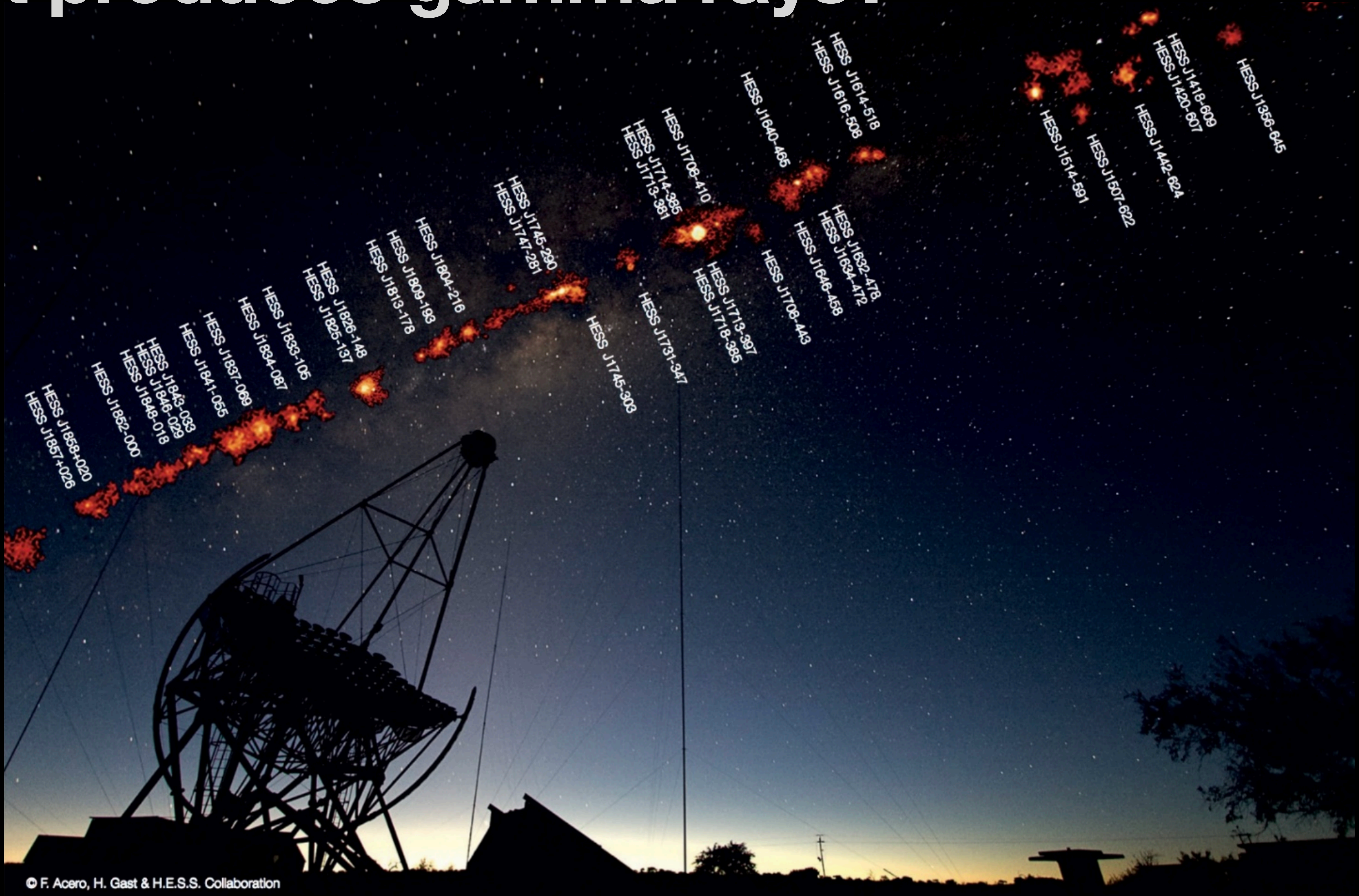
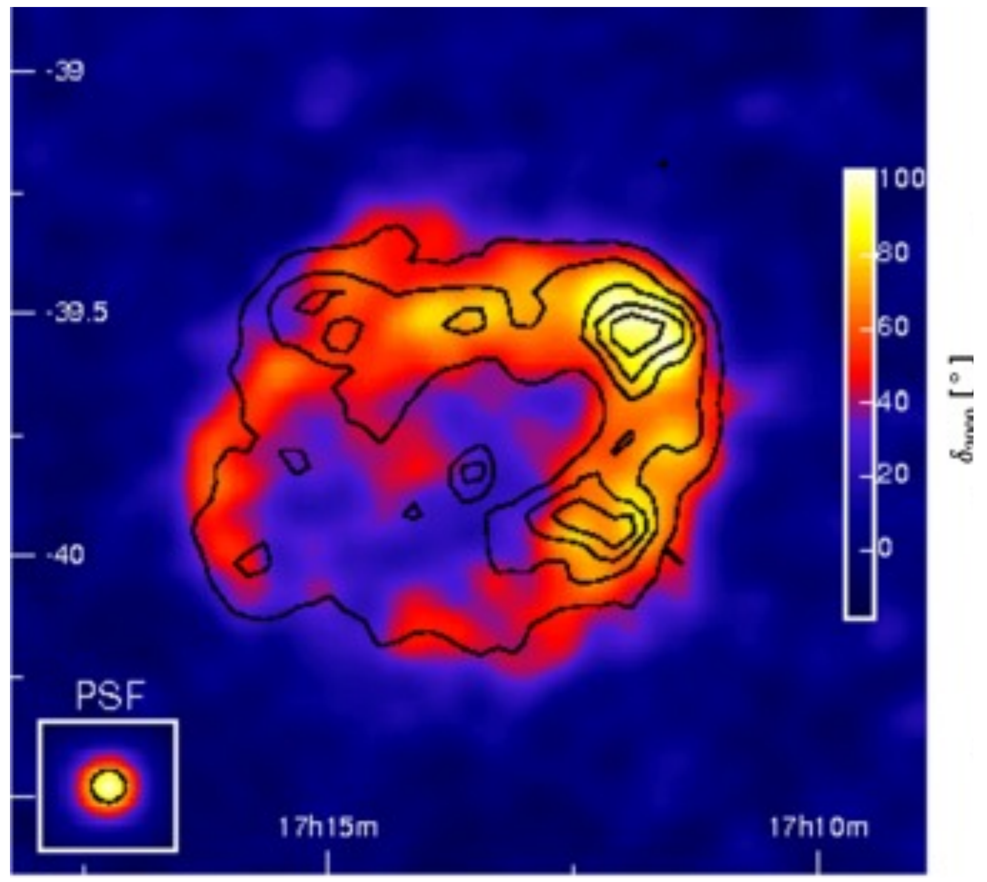
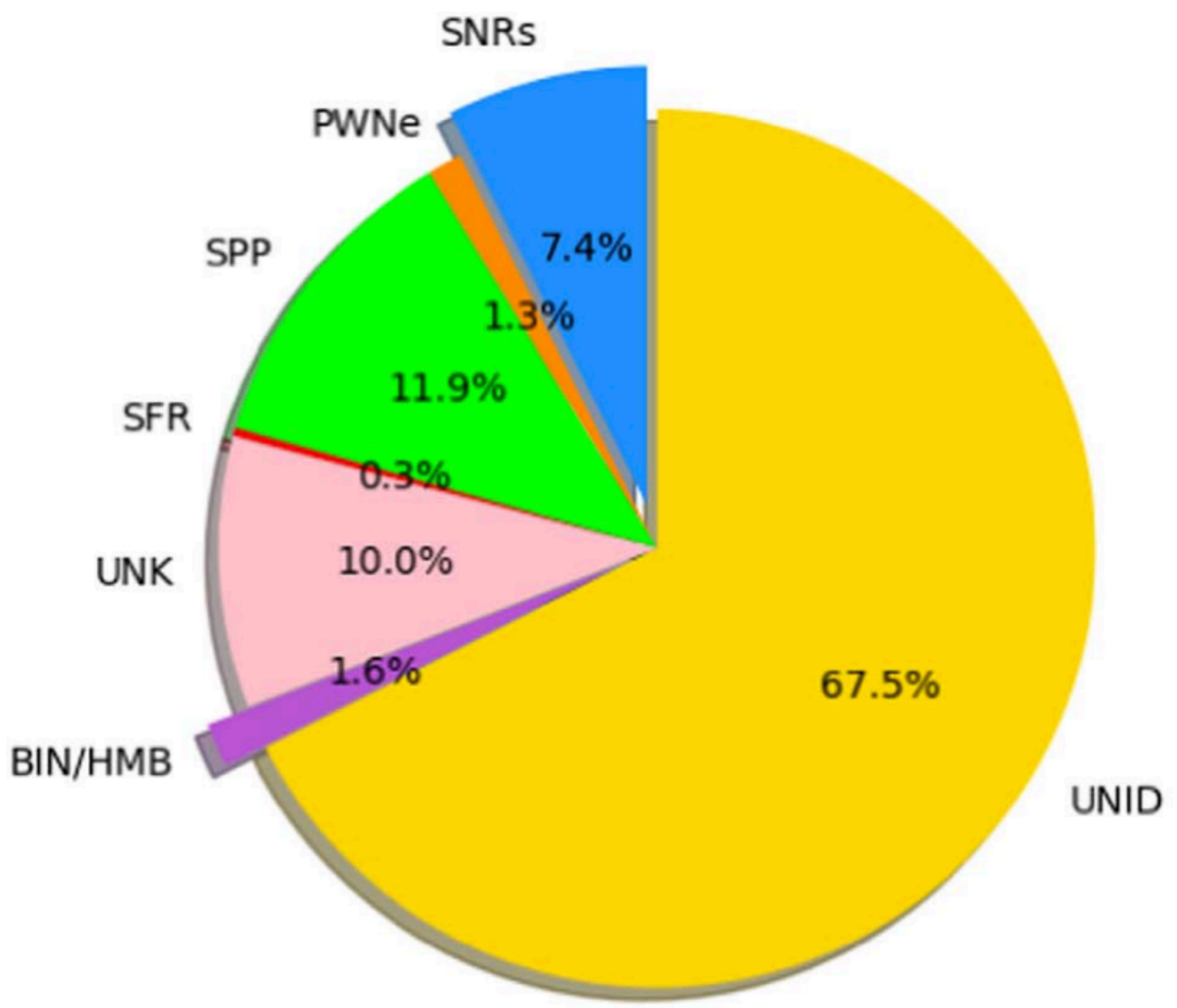


What produces gamma rays?

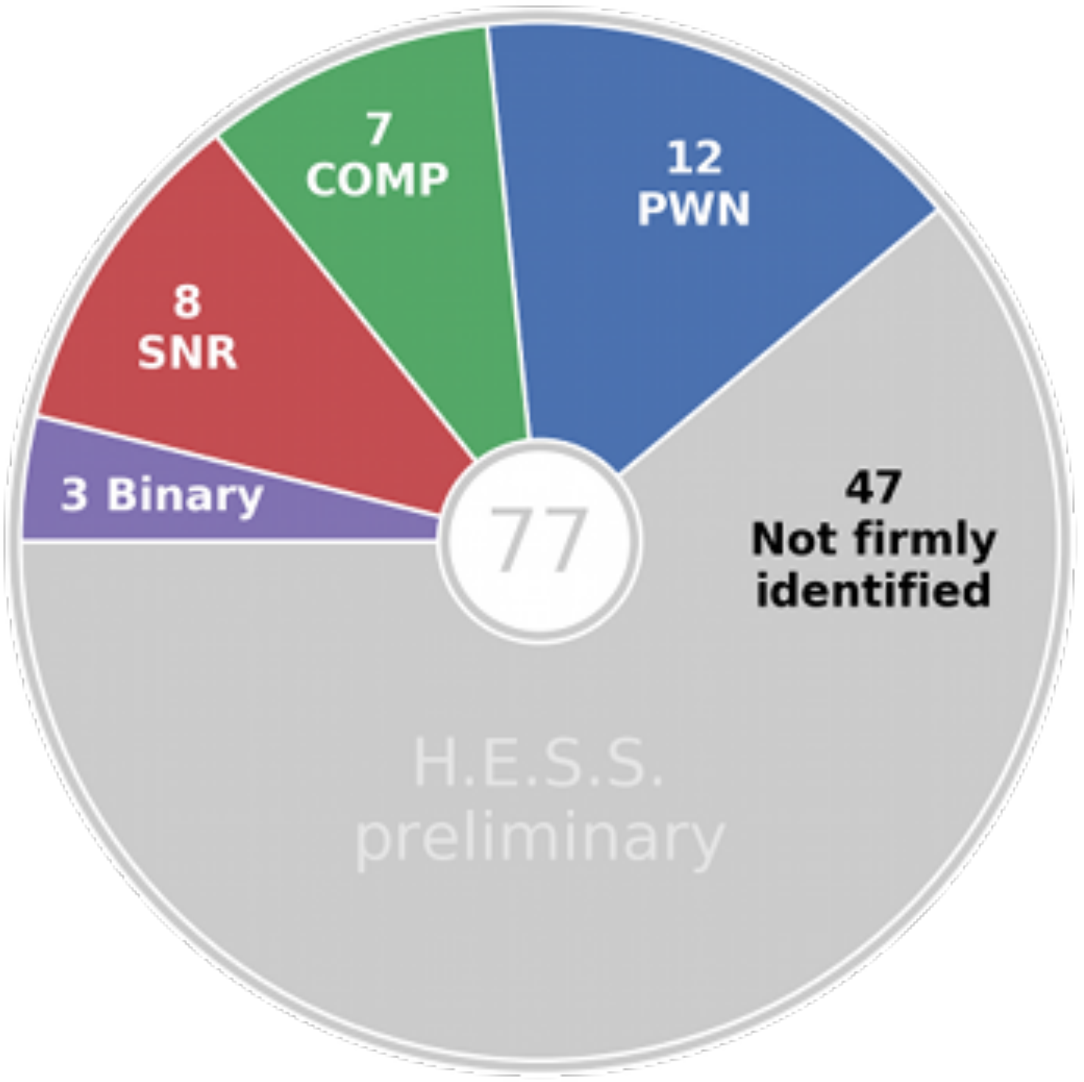


Catalogs of gamma-ray sources include SNRs, PSRs, PWNe, SCs, XRB, AGNs…

…BUT: <u>Gamma-ray sources are NOT producing gamma rays!</u>



Fermi-LAT (4FGL)



H.E.S.S. (HGPS)

They are *accelerating* particles and we measure by-products of the interactions of these with matter and electro-magetic fields.

What produces gamma rays?

Gamma-ray sources are NOT producing gamma rays!

***Particles* are producing gamma-ray emission**

Hadronic particles: mainly protons (but also other nuclei) interacting with matter and photons.

Leptonic particles: mainly electrons and positrons interacting with matter, photons, magnetic fields.

Dark matter particles ?? possible annihilation or decay will produce gamma ray emission

What we will see today

- **Hadronic interactions:**
    - The threshold for production of mesons
    - The pion bump and other features to recognize hadronic emission
    - Applications
- **Leptonic interactions**
    - Radiation of a charged particle (bremsstrahlung and synchrotron)
    - Inverse Compton scattering
    - Electron cooling
    - Applications

What we will not see but still is important

- **Absorption mechanisms**
  - **Self absorption:** at low frequencies affects radio and thermal bremsstrahlung influencing the shape of the spectrum;
  - **Gamma-gamma** absorption: at high energies gamma-ray photons interact with radiation fields producing pairs. Set the horizon for VHE observations.

- **Gamma-ray lines:** nuclear de-excitation lines of elements (mostly relevant at MeV energies) — decay/annihilation lines of dark matter may appear at higher energies e.g. Siegert 2026 https://link.springer.com/article/10.1007/s11214-026-01281-y.

Hadronic interactions

Hadronic interactions

Energetic nuclei impact on ambient nuclei or photons : pp /pγ

$$p(A) + p(A,\gamma) \rightarrow \pi^{0,\pm} + X \begin{cases} \pi^0 \rightarrow 2\gamma \\ \pi^+ \rightarrow \mu^+ + \nu_\mu \\ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \end{cases}$$

Interaction increased when high target (gas/radiation) density

Hadronic interactions

Energetic nuclei impact on ambient nuclei or photons :

pp /pγ

The interaction produces unstable mesons (mostly pions but also also η, ω)

$$p(A) + p(A,\gamma) \to \pi^{0,\pm} + X \begin{cases} \pi^0 \to 2\gamma \\ \pi^+ \to \mu^+ + \nu_\mu \\ \pi^- \to \mu^- + \bar{\nu}_\mu \end{cases}$$

The process requires a minimum energy: threshold for producing pions

Hadronic interactions

Energetic nuclei impact on ambient nuclei or photons :

pp /pγ

The interaction produces unstable mesons (mostly pions but also also η, ω)

The pions quickly decay into gamma rays and neutrinos

$$p(A) + p(A,\gamma) \rightarrow \pi^{0,\pm} + X \begin{cases} \pi^0 \rightarrow 2\gamma \\ \pi^+ \rightarrow \mu^+ + \nu_\mu \\ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \end{cases}$$

Decay time of pions:

$$\tau(\pi^0) = 8.4 \times 10^{-17}\text{s}$$
$$\tau(\pi^\pm) = 2.6 \times 10^{-8}\text{s}$$

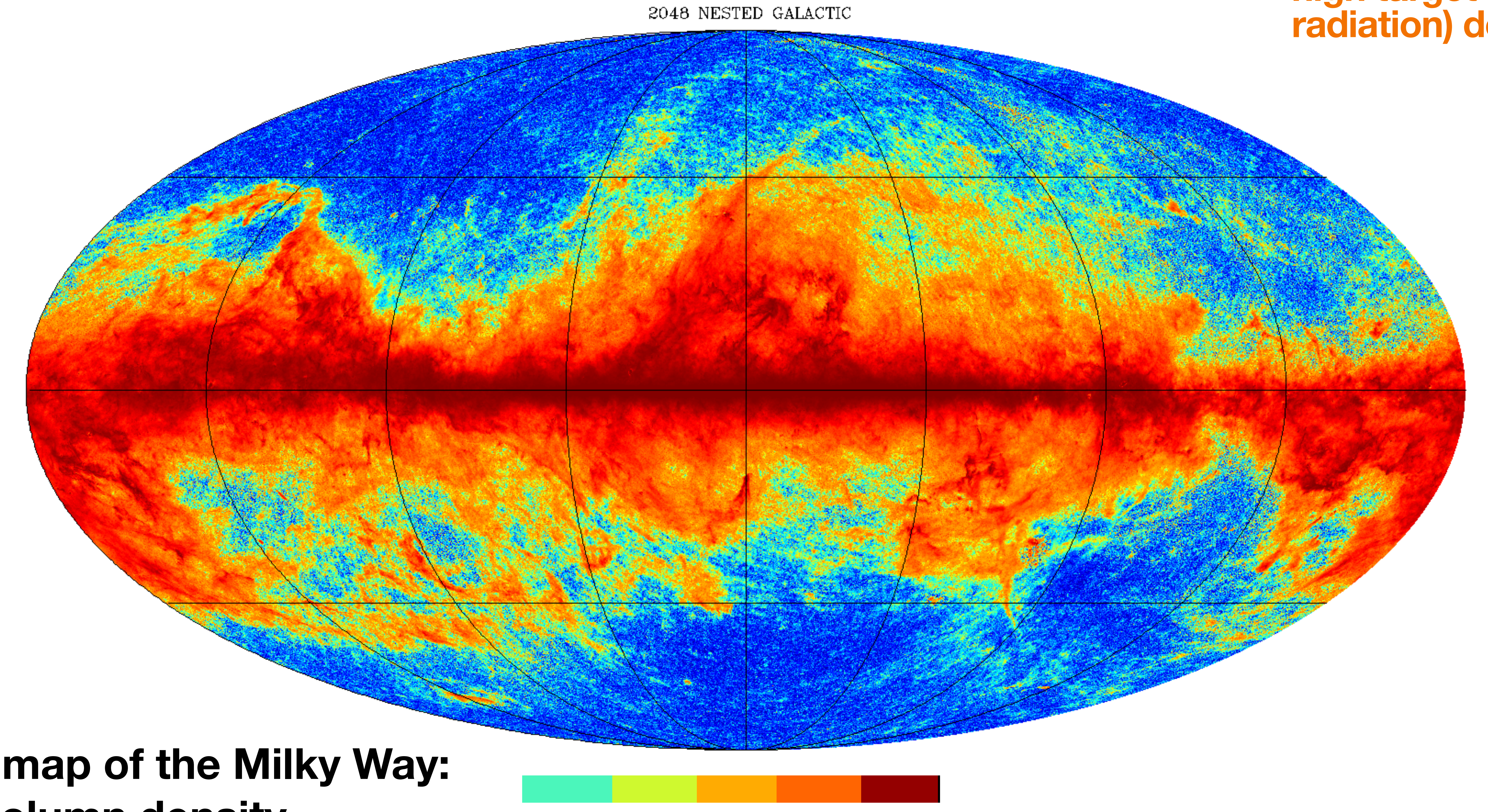
Gamma rays and neutrinos come from the interaction region

Hadronic interactions

$$p(A) + p(A,\gamma) \rightarrow \pi^{0,\pm} + X$$

Interaction increased when high target (gas/radiation) density

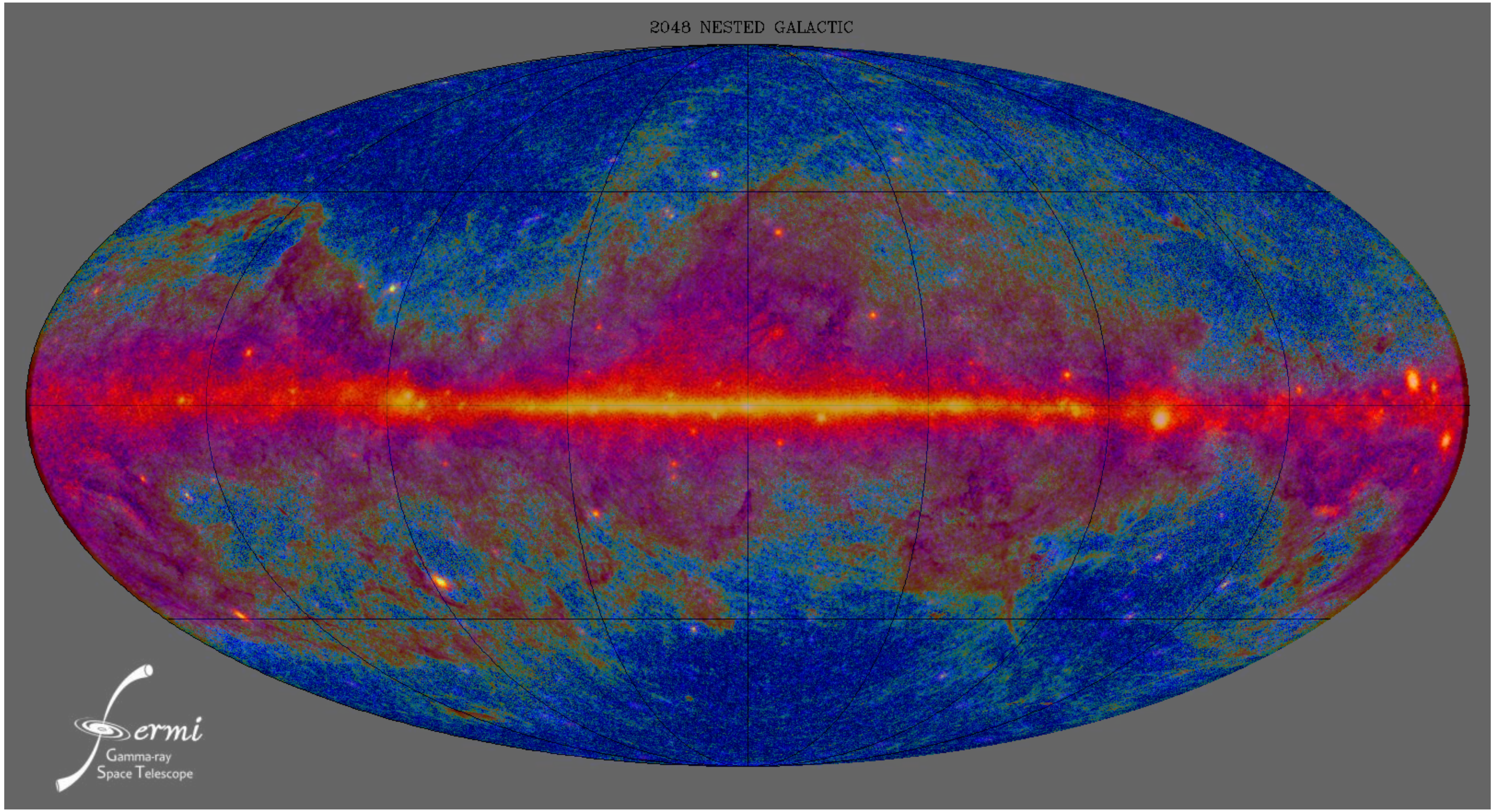
Where are they important?



Planck dust map of the Milky Way: traces gas column density

Hadronic interactions

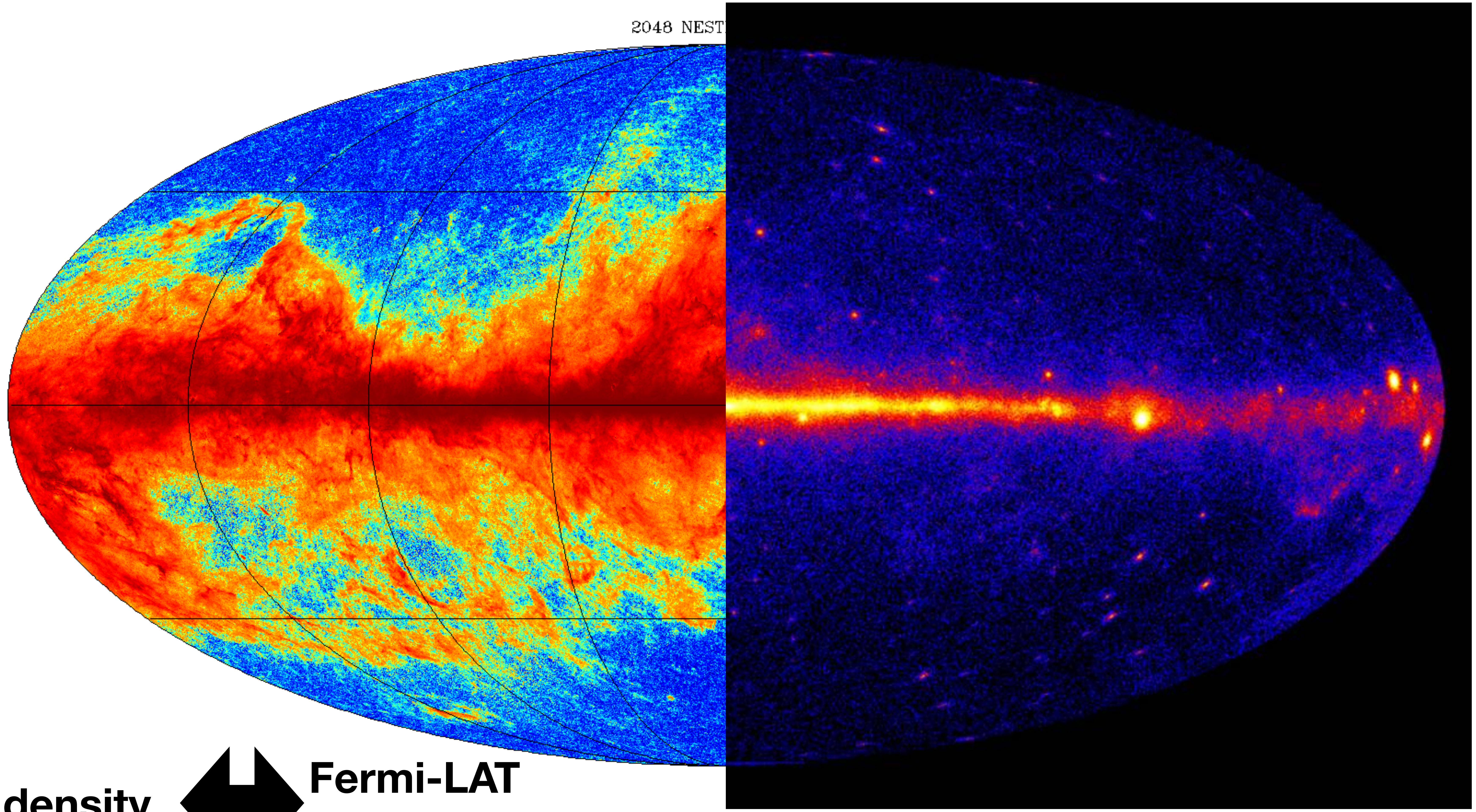
Where are they important?



Fermi-LAT > 1 GeV emission

Hadronic interactions

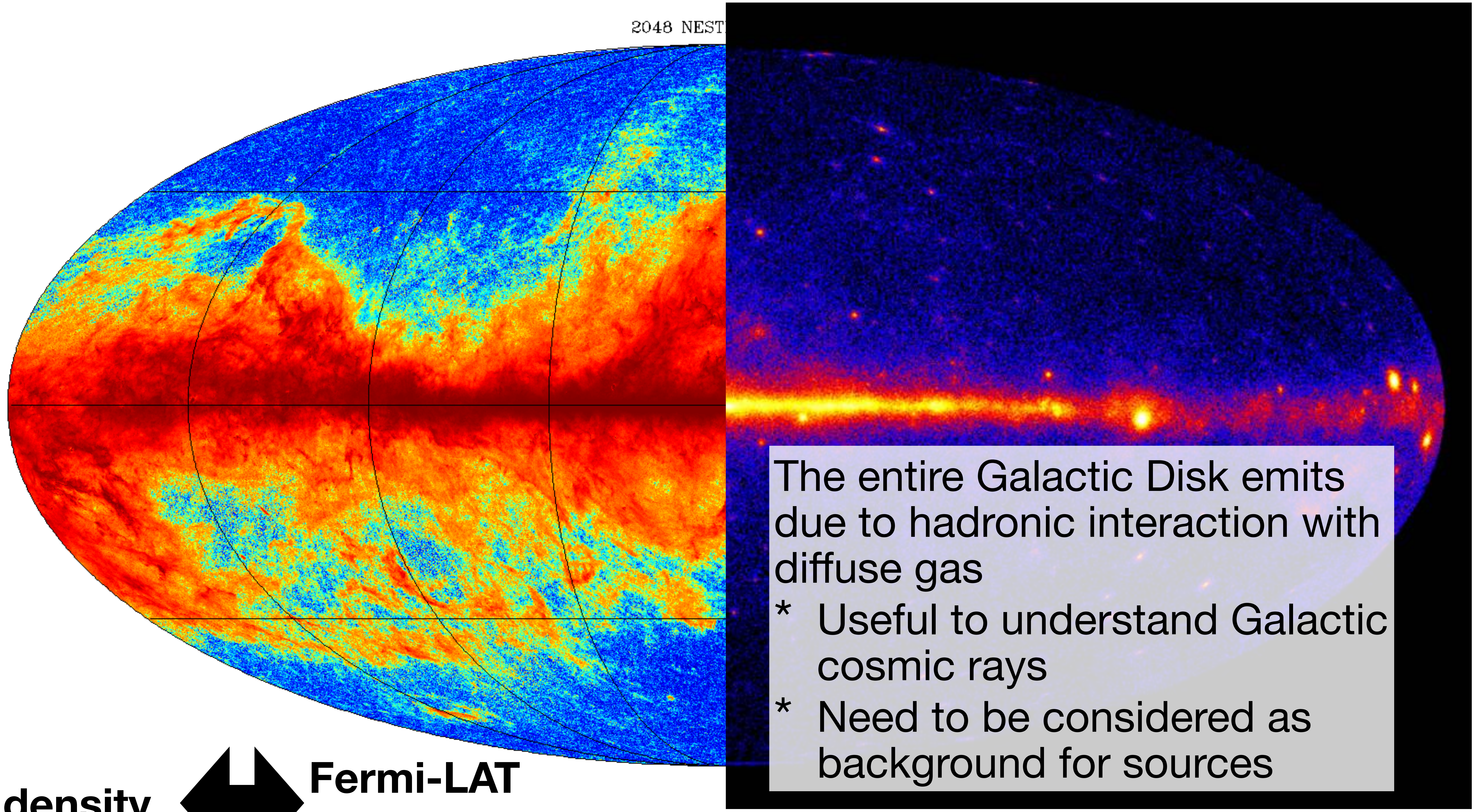
Where are they important?



Gas column density ⟷ **Fermi-LAT**
> 1 GeV emission

Hadronic interactions

Where are they important?



The entire Galactic Disk emits due to hadronic interaction with diffuse gas

- Useful to understand Galactic cosmic rays
- Need to be considered as background for sources

Gas column density ⟷ **Fermi-LAT > 1 GeV emission**

Pion decay (pp)

Threshold of the mechanism

$$p + p \to p + p + \pi^0$$

$$\boxed{s_{th} = (\sum M_i c^2)^2} = (\underbrace{2m_p c^2}_{\text{2 protons}} + \underbrace{m_{\pi^0} c^2}_{\text{1 pion}})^2$$

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$\sqrt{s}$ total energy in the center of mass

s is invariant per reference system

In the lab. frame:

$$p_1^\mu = (E_p, 0, 0, pc)$$ One proton is moving along z

$$p_2^\mu = (m_p c^2, 0, 0, 0)$$ One proton is at rest

$$(p_1 + p_2)^2 = (E_p + m_p)^2 - p^2$$

Metric $(1, -1, -1, -1)$

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2 protons 1 pion

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Metric $(1, -1, -1, -1)$

$$E_p^2 - (pc)^2 = (m_p c^2)^2$$

$$s = (p_1 + p_2)^2 \rightarrow s_{lab} = (2m_p^2 c^4 + 2m_p c^2 E_p)$$

$$\rightarrow s_{lab} = s_{th} \rightarrow (2m_p c^2 + m_{\pi^0} c^2)^2 = (2m_p^2 c^4 + 2m_p c^2 E_p)$$

$$E_p^K = E_p - m_p c^2$$

$$E_p^{K,\mathrm{th}} = 2m_{\pi^0} c^2 + \frac{m_{\pi^0}^2 c^2}{2m_p} = \boxed{280\ \text{MeV}}$$

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Exercise: how much does the threshold change for the production of π^+ ?

Pion decay (p-γ)

Threshold of the mechanism

$$p + \gamma \rightarrow p + \pi^0$$

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1 proton 1 pion

In the lab. frame:

$$p_p^\mu = (E_p, 0, 0, p_p c)$$

The proton is moving along z

$$p_\gamma^\mu = (\epsilon_\gamma, 0, 0, p_\gamma c)$$

$$p_\gamma = \frac{\epsilon_\gamma}{c}$$

θ is the angle between proton and photon

$$s_{lab} = (p_p + p_\gamma)^2 = (m_p c)^2 + 2 p_p \epsilon (1 - \cos\theta)$$

$$p_p \approx \frac{E_p}{c}; E_p \gg m_p c^2$$

$$s_{lab} = (m_p c^2)^2 + 2 E_p \epsilon (1 - \cos\theta)$$

$$s_{th} = (m_p c^2 + m_{\pi^0} c^2)^2$$

Pion decay (p-γ)

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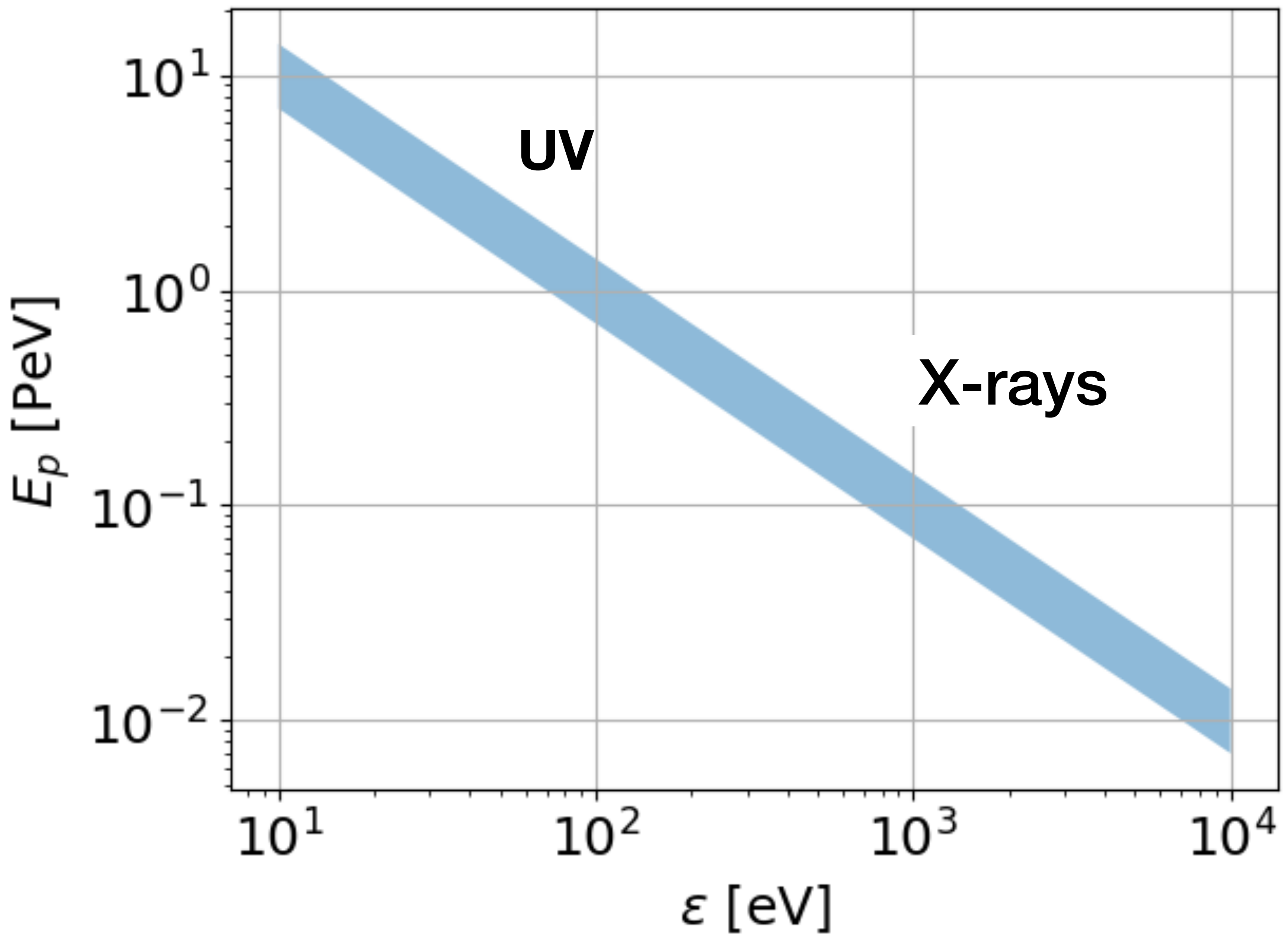

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$$s_{lab} = (m_p c^2)^2 + 2 E_p \epsilon (1 - \cos\theta)$$

$$s_{th} = (m_p c^2 + m_{\pi^0} c^2)^2$$

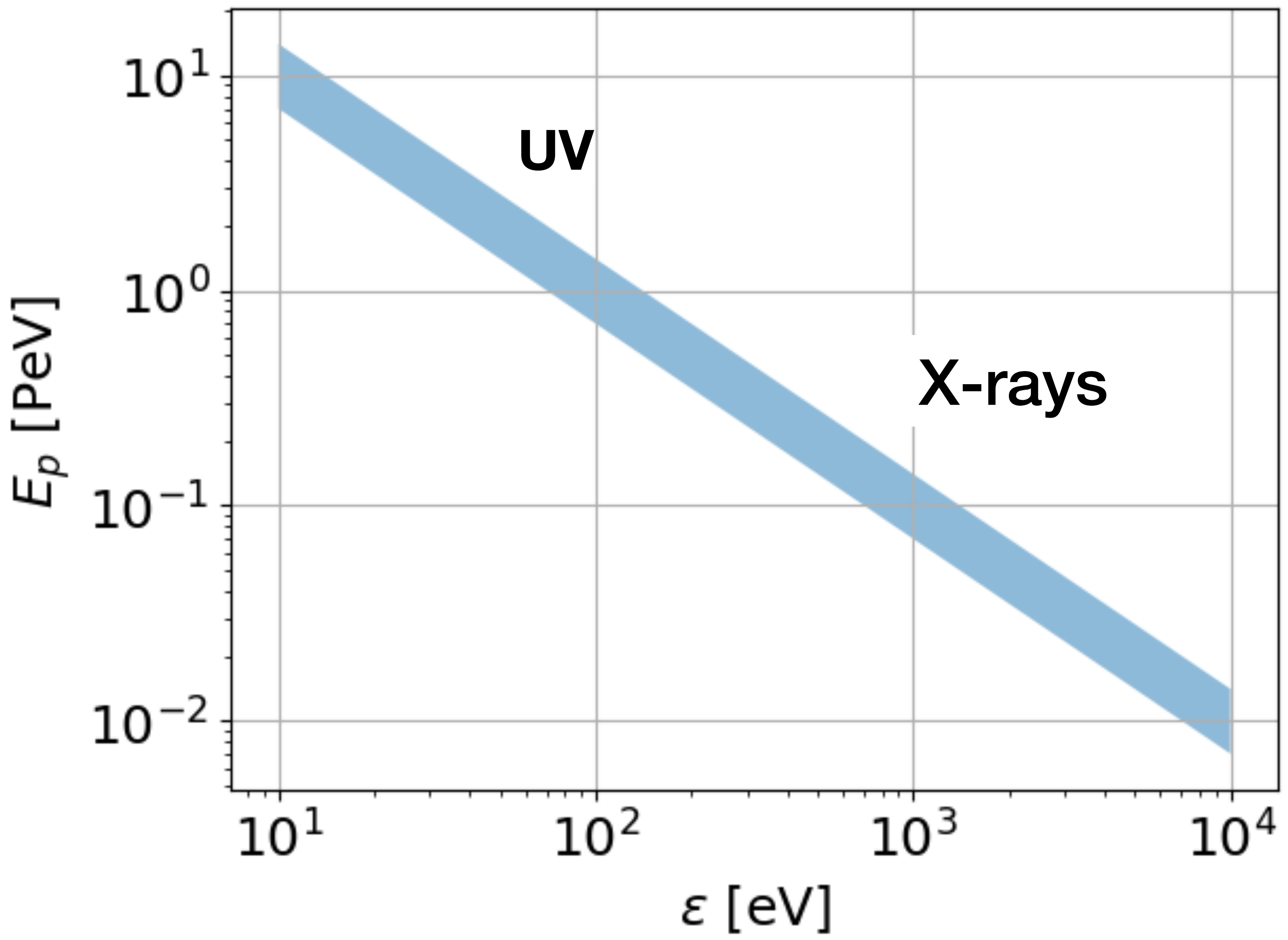
$$\Rightarrow E_p^{th} = \frac{2 m_p m_{\pi^0} + m_{\pi^0}^2}{2\epsilon(1 - \cos\theta)} = \frac{\boxed{140 \text{ PeV}}}{(1 - \cos\theta)\frac{\epsilon}{1 \text{ eV}}}$$

Pion decay (p-γ)

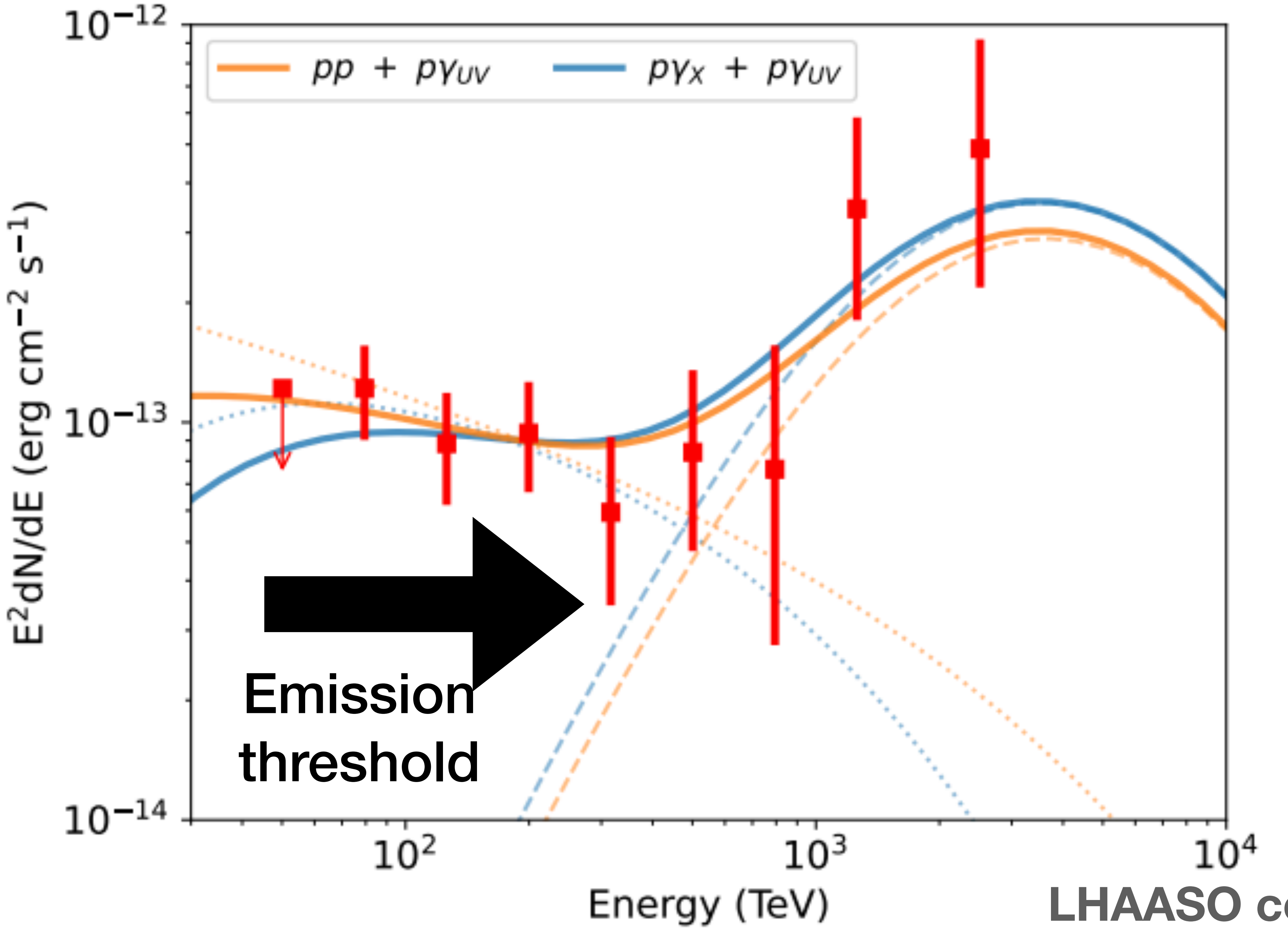
Threshold of the mechanism

$$p + \gamma \rightarrow p + \pi^0$$

Emission appears at Very/Ultra high energies



Cygnus-X3 micro-quasar: have large X-ray and UV field



LHAASO coll. 2026

$$E_p^{th} = \frac{2m_p m_{\pi^0} + m_{\pi^0}^2}{2\epsilon(1-\cos\theta)} = \frac{\boxed{140 \text{ PeV}}}{(1-\cos\theta)\frac{\epsilon}{1 \text{ eV}}}$$

From protons to pions

We understood the threshold for producing pions, but how much energy is transferred to the pions?

How is the resulting spectrum of pions?

From protons to pions

In the δ approximation

..we assume that a fixed fraction of energy k_π is transferred from protons to pions in the collision

Pion emissivity

$$q_\pi(E_\pi) = cn_H \int \delta(E_\pi - k_\pi E_{kin}) \sigma_{pp}(E_p) N_p(E_p) dE_p$$

Fraction of transferred energy from proton to pion ~17%

inelastic cross section

Energy distribution of protons

$$\left[E_p = m_p c^2 + \frac{E_\pi}{k} \right]$$

$$= \frac{cn_H}{k_\pi} \sigma_{pp}\left(m_p c^2 + \frac{E_\pi}{k_\pi} \right) N_p\left(m_p c^2 + \frac{E_\pi}{k_\pi} \right)$$

From protons to pions

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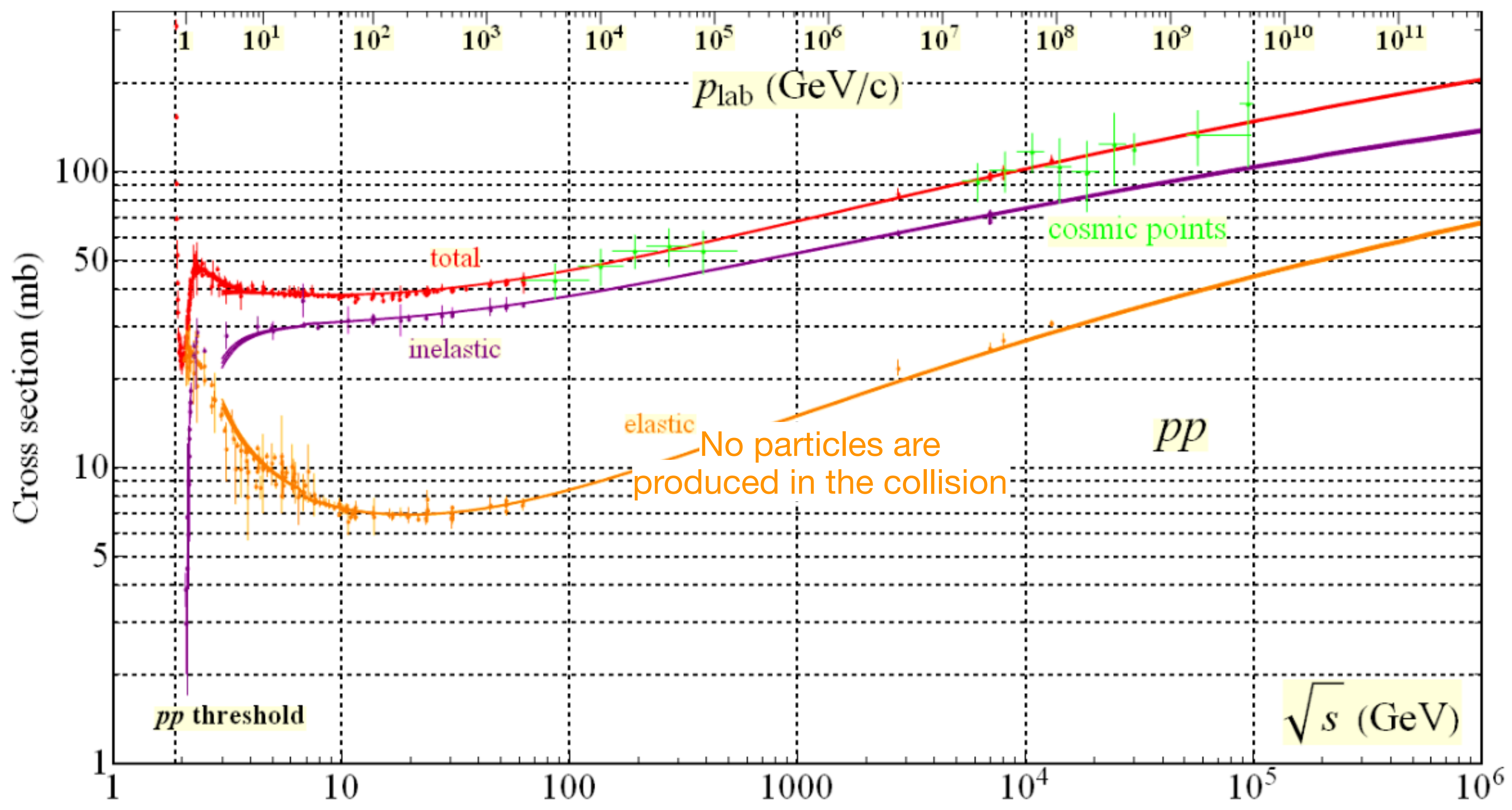
Kelner & Aharonian approximation (>2 GeV) :

$$\sigma_{pp}(E_p) \approx 30\left[0.95 + 0.06 \ln\left(\frac{E_{kin}}{1 \text{ GeV}}\right)\right]$$

Total cross section

Laboratory measurement



Slight dependence on energy: Spectral shape of pions will follow the shape of protons

From protons to pions

In the δ approximation

$$q_\pi(E_\pi) = cn_H \int \delta(E_\pi - k_\pi E_{kin})\sigma_{pp}(E_p)N_p(E_p)dE_p$$

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inelastic cross section

Energy distribution of protons

$$= \frac{cn_H}{k_\pi}\sigma_{pp}\left(m_pc^2 + \frac{E_\pi}{k_\pi}\right)N_p\left(m_pc^2 + \frac{E_\pi}{k_\pi}\right)$$

Kelner & Aharonian approximation (>2 GeV) :

$$\sigma_{pp}(E_p) \approx 30\left[0.95 + 0.06\ln\left(\frac{E_{kin}}{1\ \mathrm{GeV}}\right)\right]\mathrm{mb}$$

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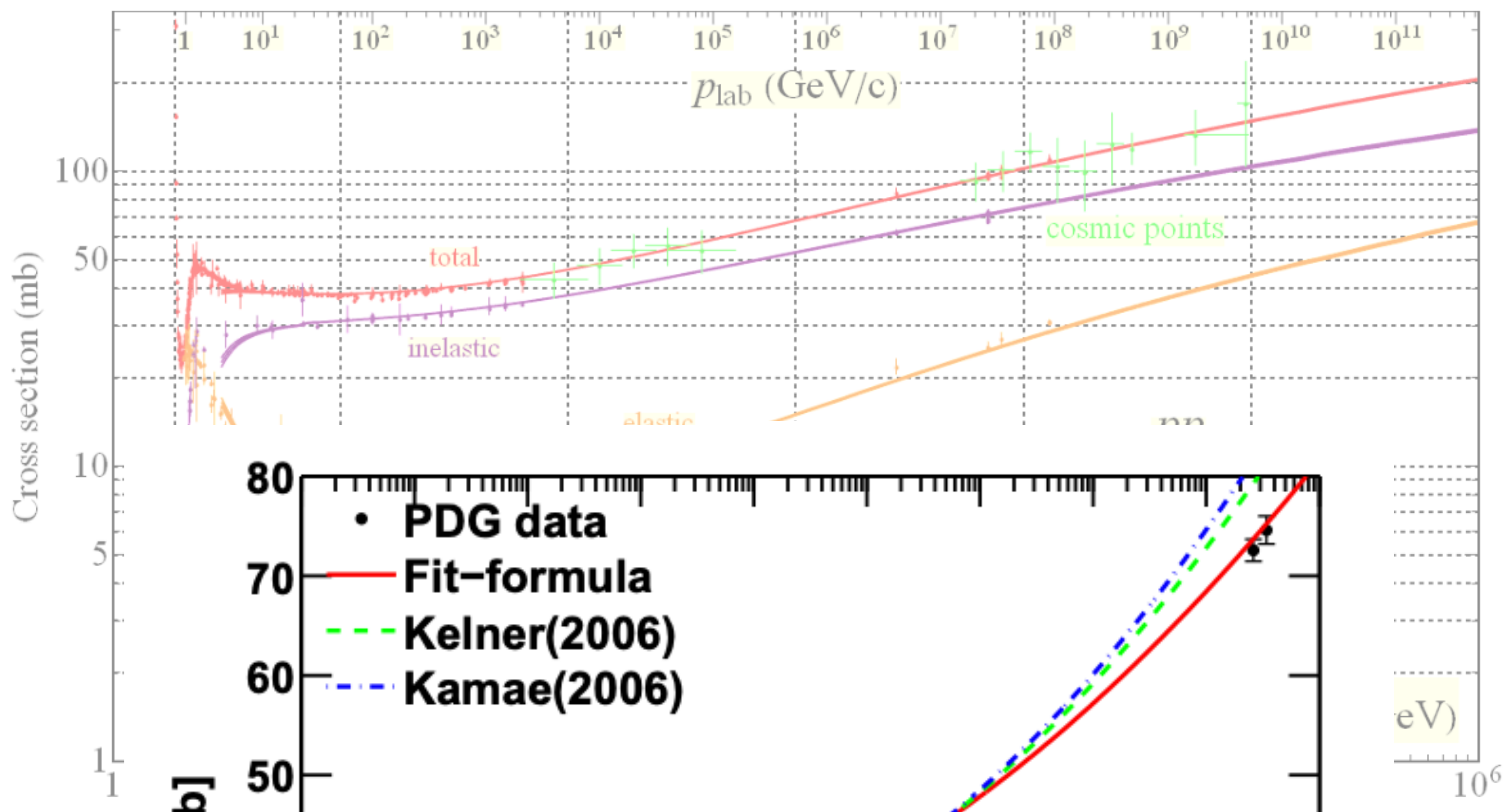
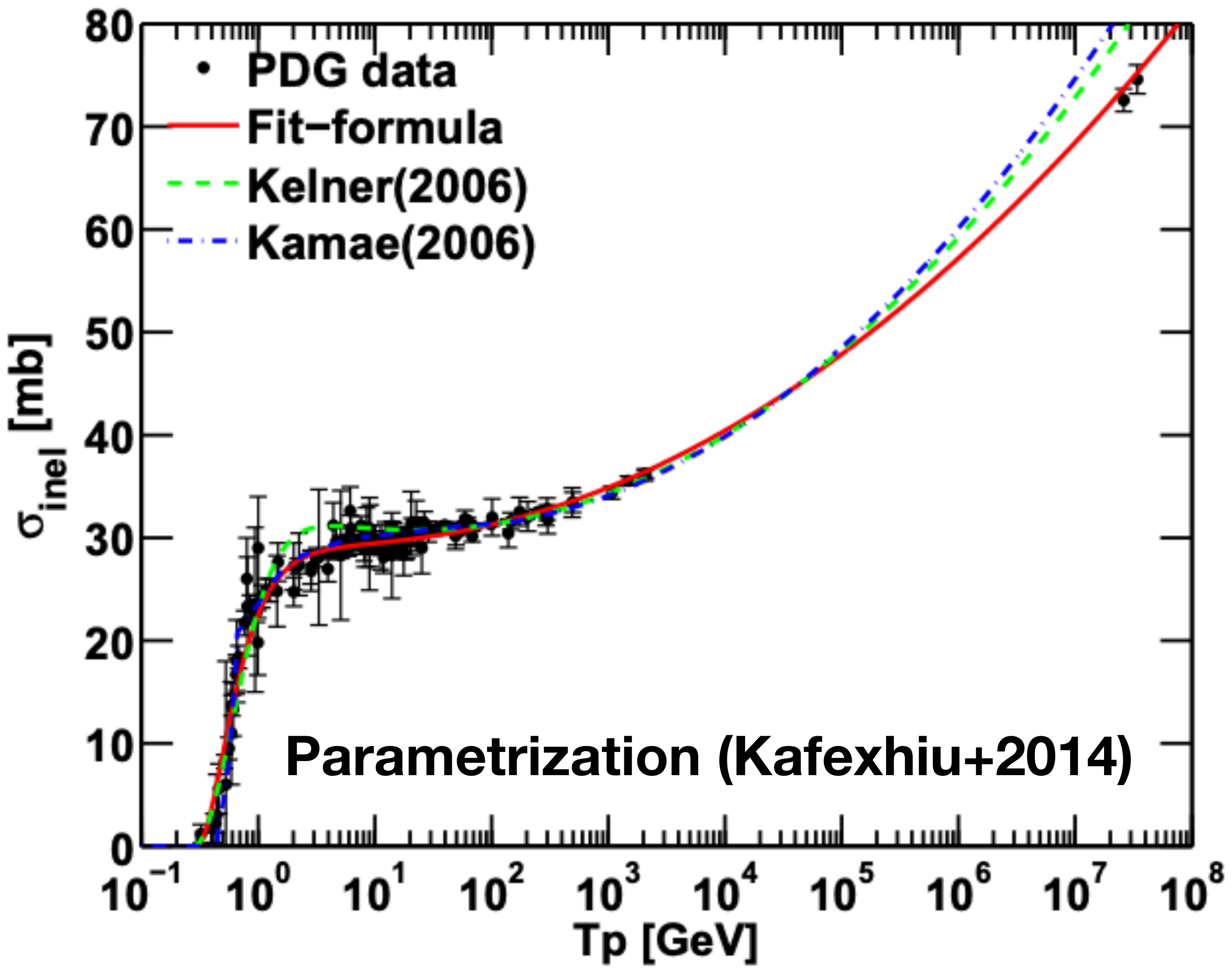




From protons to pions

Total inelastic cross section and cooling time

$$\sigma_{pp}(E_p) \approx 30\left[0.95 + 0.06\ln\left(\frac{E_{kin}}{1\ \mathrm{GeV}}\right)\right]\mathrm{mb}$$

Gives the probability of interaction hence the mean free path, hence the time between each collision

$$t_{pp} = (n\sigma_{PP}c)^{-1} \simeq 5.4\times 10^{7}\left(\frac{n}{\mathrm{cm}^{-3}}\right)^{-1}\mathrm{yr}$$

~ residence time in the MW of CRs of 10-20 Myr derived from unstable isotopes

Proton may travel quite some times before interacting and emitting!

From protons to pions

In the δ approximation

$$q_\pi(E_\pi) = c n_H \int \delta(E_\pi - k_\pi E_{kin}) \sigma_{pp}(E_p) N_p(E_p) dE_p$$

Fraction of transferred energy from proton to pion ~17%

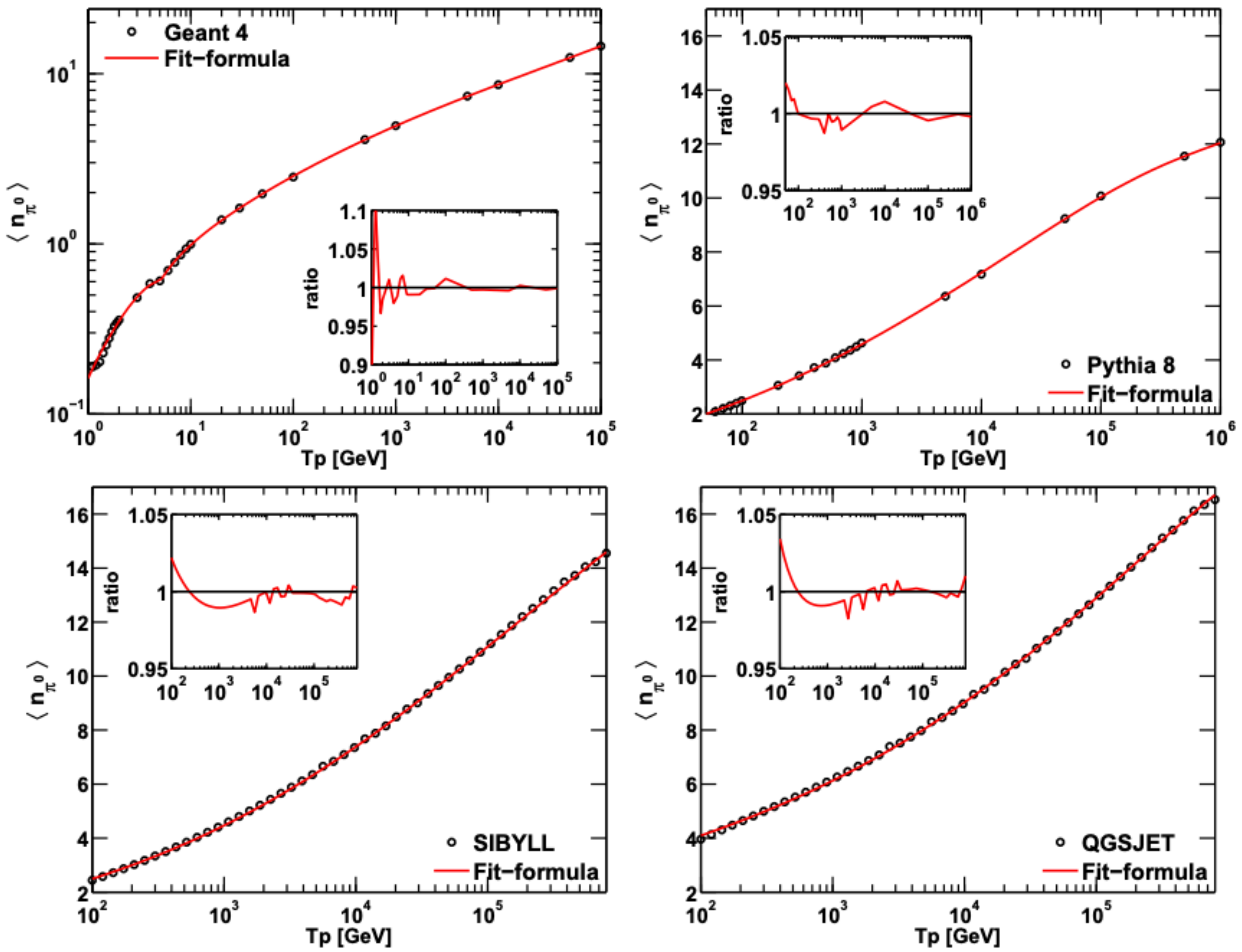
inelastic cross section

Energy distribution of protons

$$= \frac{c n_H}{k_\pi} \sigma_{pp}\left(m_p c^2 + \frac{E_\pi}{k_\pi}\right) N_p\left(m_p c^2 + \frac{E_\pi}{k_\pi}\right)$$

1st complication: Multiplicity

How many pions are produced per interaction (multiplicity) depends on the energy in the center of mass: the more energy the higher number of pions are produced



Kafexhiu+2014

From protons to pions

Beyond the δ approach

$$q_\pi(E_\pi) = c n_H \int \cancel{\delta(E_\pi - k_\pi E_{kin})} \sigma_{pp}(E_p) N_p(E_p) dE_p$$

$$q_\pi(E_\pi) = c n_H \int F_\pi(x, E_p) \sigma_{pp}(E_p) N_p(E_p) dE_p$$

2nd complication: the transferred energy is not constant

$$dN_\pi = F_\pi dx$$

$$x = \frac{E_\pi}{E_p}$$

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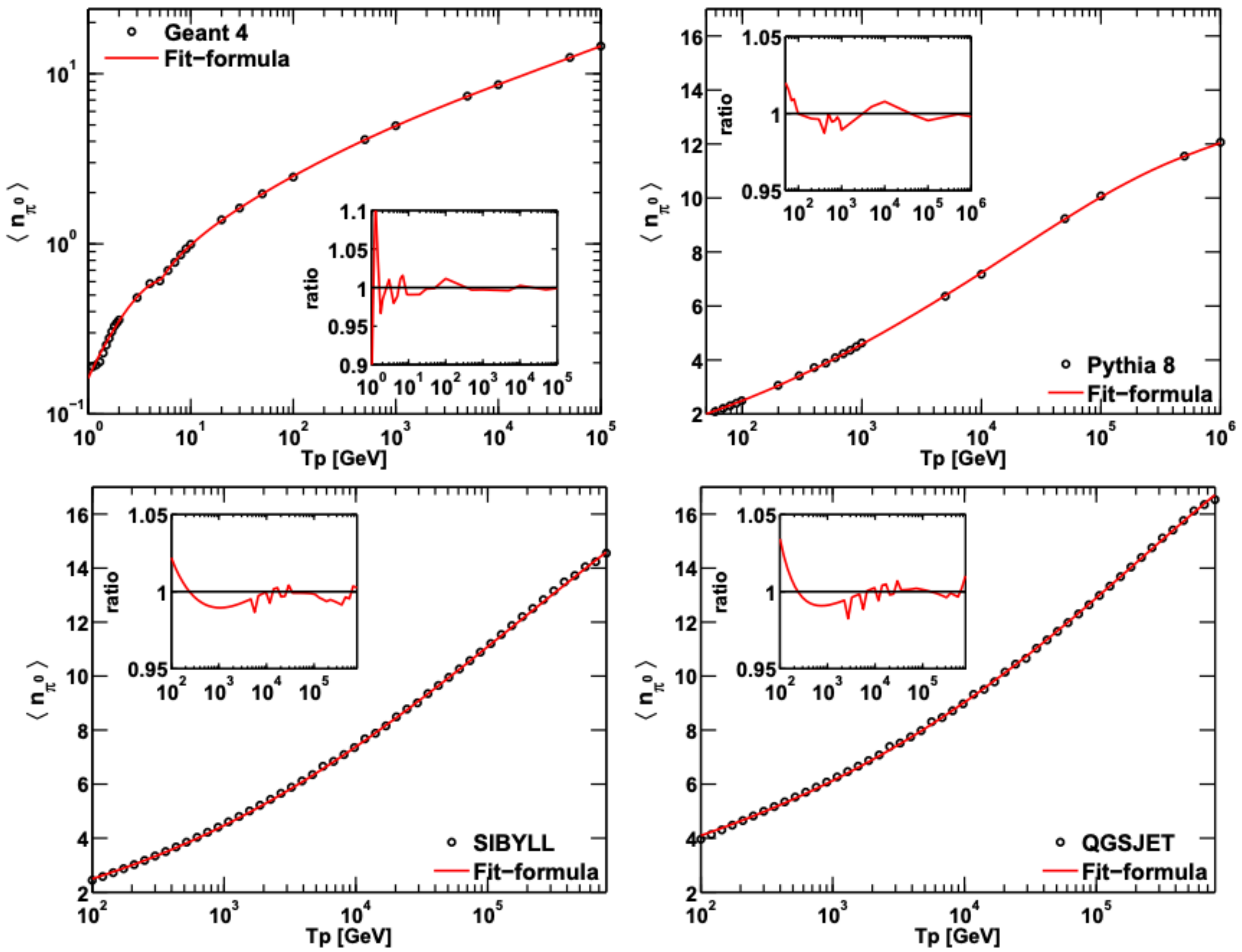

From protons to pions

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To compute the final pion spectrum one needs a numerical approach

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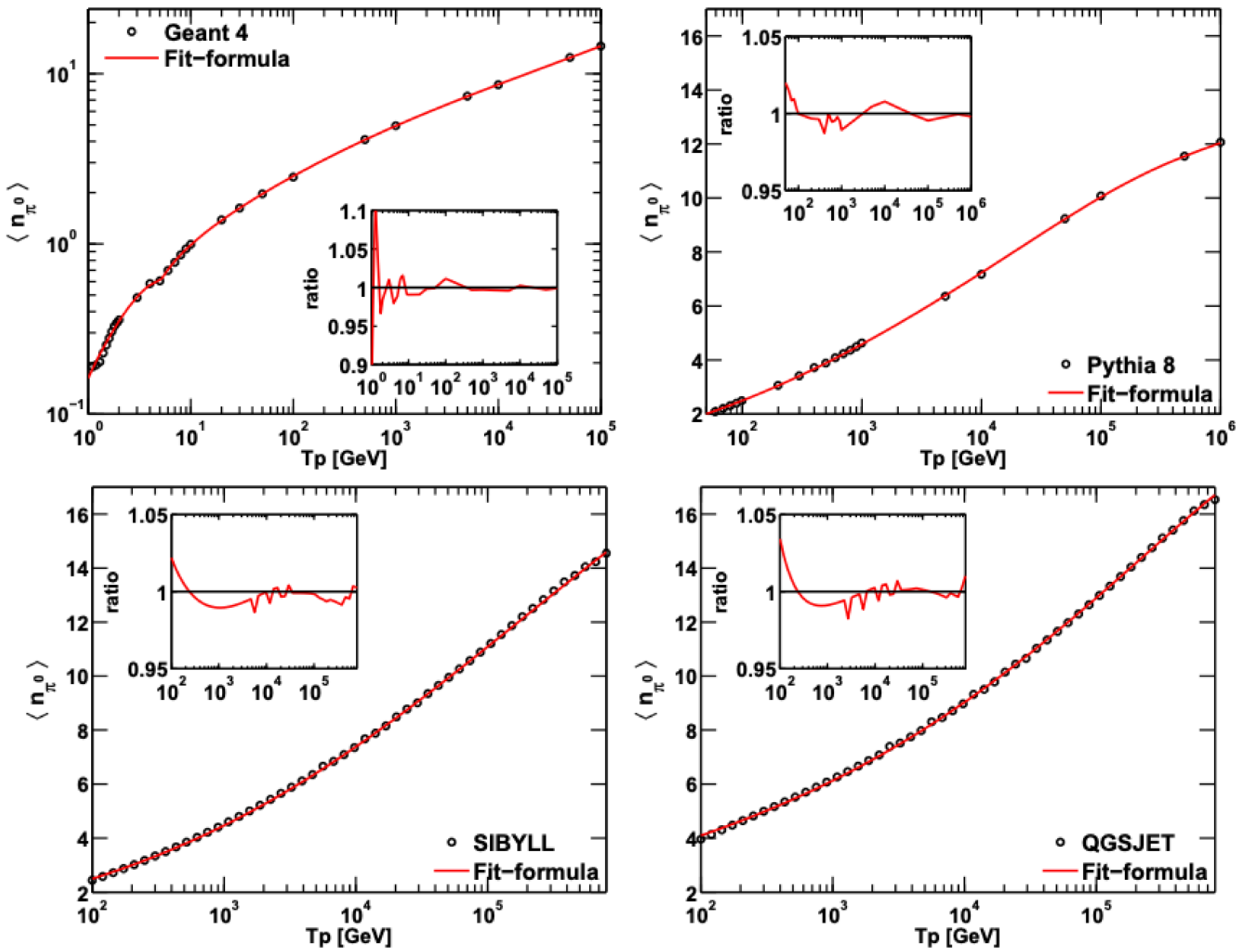

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$$dN_\pi = F_\pi dx$$

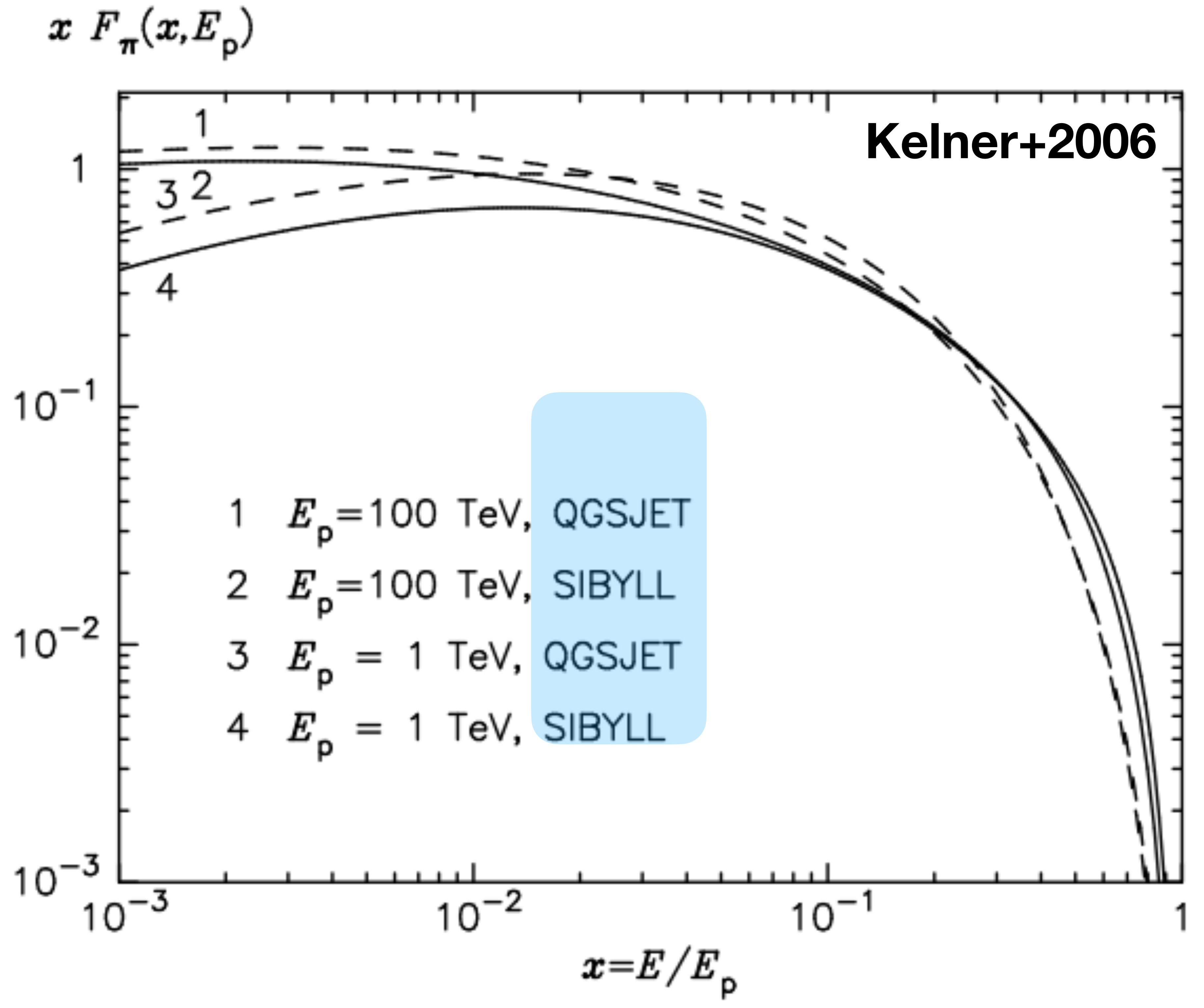
$$x = \frac{E_\pi}{E_p}$$

Fit to the generated data **(Kelner+2006)**

$$F_\pi(x,\,E_p) = 4\,\alpha B_\pi x^{\alpha-1}\left(\frac{1-x^\alpha}{(1+r\,x^\alpha)^3}\right)^4 \times \left(\frac{1}{1-x^\alpha}+\frac{3\,r}{1+rx^\alpha}\right)\left(1-\frac{m_\pi}{x\,E_p}\right)^{1/2}.$$

Energy spectra of pions

Taking into account multiplicity and energy transfer, as derived with numerical approaches like (QGSJET, SIBYLL..)



From pions to photons

The shape of photon spectrum

In the rest frame of pion photons are emitted isotropically

$$\pi^0 \rightarrow 2\gamma$$

This is a consequence of the fact that pions have spin J=0

Now we have to compute how much energy is transferred from the pion to the photons in the decay. We follow F. Stecker derivation.

From pions to photons

The shape of photon spectrum: the pion bump

$$\beta = \sqrt{1 - \frac{1}{\Gamma^2}}$$

$$\Gamma = \frac{E_\pi}{m_{\pi^0} c^2} = \frac{\eta E_p}{m_{\pi^0} c^2}$$

In the rest frame of pion photons are emitted isotropically

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Energy of the photon in the

Pion rest frame: $E^*_\gamma = \frac{m_{\pi^0} c^2}{2}$

Lab. frame: $E_\gamma = \Gamma E^*_\gamma (1 + \beta \cos \theta^*) \rightarrow -1 < \cos \theta^* < 1$

There is a maximum and a minimum

θ^* is the angle of emission of photons in the pion rest frame

From pions to photons

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$$\begin{cases} E_{\gamma,min} < E_\gamma < E_{\gamma,max} \\ E_{\gamma,min} = \Gamma E*_\gamma(1-\beta) = E_\gamma(E_\pi) \\ E_{\gamma,max} = \Gamma E*_\gamma(1+\beta) = E_\gamma(E_\pi) \end{cases}$$

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Photon distribution for each pion of energy E_π

$$f(E_\gamma) = \frac{dN^j_\gamma}{dE_\gamma} = \frac{1}{E_{max} - E_{min}} = \frac{1}{\Gamma m_{\pi^0}\beta} = \frac{1}{\sqrt{E_\pi^2 - m_{\pi^0}^2}}$$

Because of isotropy: each photon is emitted with the same probability between E_{min} and E_{max}

From pions to photons

The shape of photon spectrum: the pion bump

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In the rest frame of pion photons are emitted isotropically

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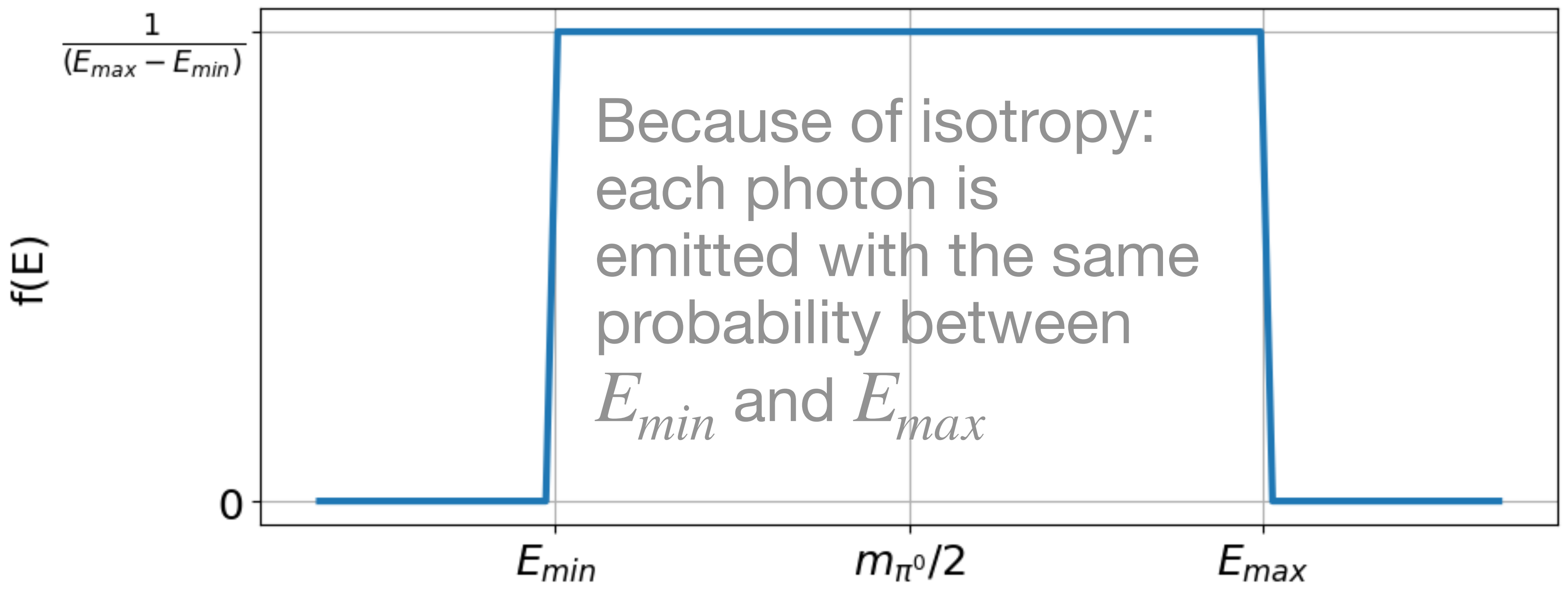
Photon distribution for each pion of energy E_π

$$f(E_\gamma) = \frac{dN^j_\gamma}{dE_\gamma} = \frac{1}{E_{max} - E_{min}} = \frac{1}{\Gamma m_{\pi^0}\beta} = \frac{1}{\sqrt{E_\pi^2 - m_{\pi^0}^2}}$$



From pions to photons

The shape of photon spectrum: the pion bump

In the rest frame of pion photons are emitted isotropically

$$\pi^0 \rightarrow 2\gamma$$

$$\beta = \sqrt{1 - \frac{1}{\Gamma^2}}$$

$$\Gamma = \frac{E_\pi}{m_{\pi^0}c^2} = \frac{\eta E_p}{m_{\pi^0}c^2}$$

Pion rest frame: $E*_\gamma = \frac{m_{\pi^0}c^2}{2}$

Lab. frame: $E_\gamma = \Gamma E*_\gamma (1 + \beta \cos\theta^*) \rightarrow -1 < \cos\theta^* < 1$

$$\begin{cases} E_{\gamma,min} < E_\gamma < E_{\gamma,max} \\ E_{\gamma,min} = \Gamma E*_\gamma (1-\beta) = E_\gamma(E_\pi) \\ E_{\gamma,max} = \Gamma E*_\gamma (1+\beta) = E_\gamma(E_\pi) \end{cases}$$

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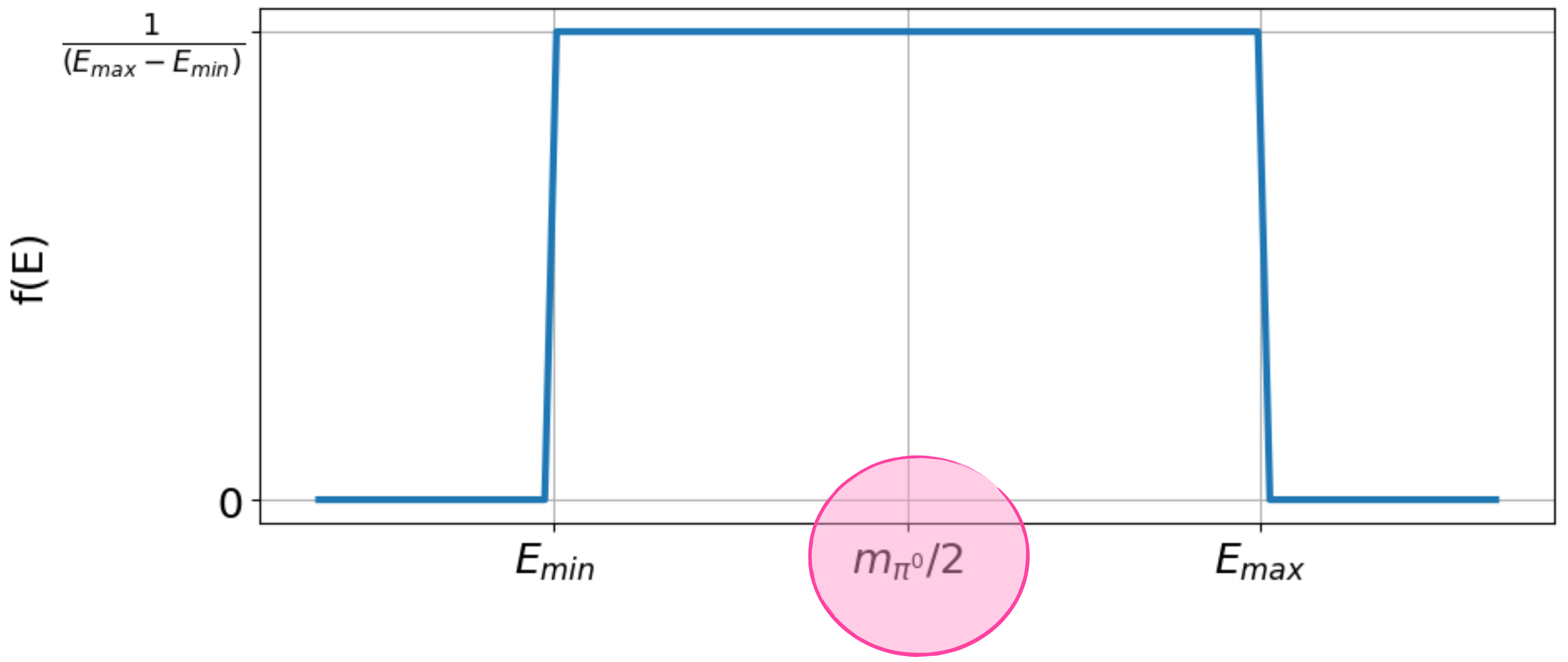

Logarithmic average between E_{min} and E_{max}

$$\log(E_{mean}) = (\log(E_{min}) + \log(E_{max}))/2$$

$$E^{log}_{mean} = (E_{min}E_{max})^{1/2} = \frac{m_{\pi^0}}{2}$$

Does not depend on E_π

From pions to photons

The shape of photon spectrum: the pion bump

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$$\pi^0 \to 2\gamma$$

$$\beta = \sqrt{1 - \frac{1}{\Gamma^2}}$$

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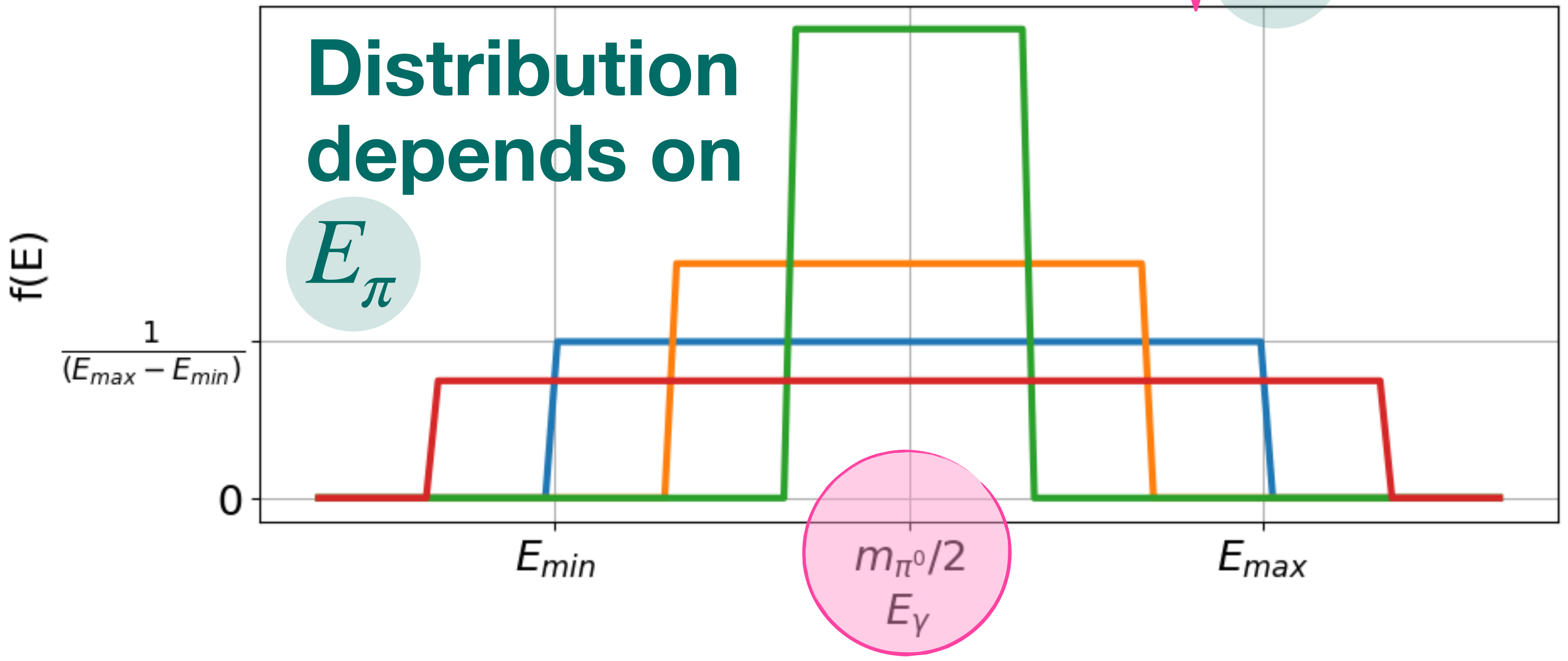

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From pions to photons

The shape of photon spectrum: the pion bump

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$$\Gamma = \frac{E_\pi}{m_{\pi^0}c^2} = \frac{\eta E_p}{m_{\pi^0}c^2}$$

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Lab. frame: $E_\gamma = \Gamma E^*_\gamma (1 + \beta \cos\theta^*) \to -1 < \cos\theta^* < 1$

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Photon distribution for each pion of energy E_π

$$f(E_\gamma) = \frac{dN^j_\gamma}{dE_\gamma} = \frac{1}{E_{max} - E_{min}} = \frac{1}{\Gamma m_{\pi^0}\beta} = \frac{1}{\sqrt{E_\pi^2 - m_{\pi^0}^2}}$$

Photon distribution for a power-law distribution of pions:

If s > 0 $\quad$ Dominated by lower boundary

$$\frac{dN_\gamma}{dE_\gamma} = \int_{E_\pi^{min}(E_\gamma)} dE_\pi \frac{dN_\pi}{dE_\pi} \frac{1}{\sqrt{E_\pi^2 - m_{\pi^0}^2}} = \int_{E_\pi^{min}(E_\gamma)} dE_\pi E_\pi^{-s} \frac{1}{\sqrt{E_\pi^2 - m_{\pi^0}^2}} \boxed{\approx (E_\pi^{min}(E_\gamma))^{-s}}$$

From pions to photons

The shape of photon spectrum: the pion bump

In the rest frame of pion photons are emitted isotropically

$$\pi^0 \rightarrow 2\gamma$$

$$\beta = \sqrt{1 - \frac{1}{\Gamma^2}}$$

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$$\Gamma = \frac{E_\pi}{m_{\pi^0}c^2} = \frac{\eta E_p}{m_{\pi^0}c^2}$$

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Photon distribution for each pion of energy E_π

$$f(E_\gamma) = \frac{dN_\gamma}{dE_\gamma} = \frac{1}{E_{Emax} - E_{min}}$$

Photon distribution for a power-law distribution of pions
(~ power-law distribution of protons):

$$\approx (E_\pi^{min}(E_\gamma))^{-s}$$

Dominated by
lower boundary

(B) $\rightarrow E_\pi^{min} = E_\gamma + \frac{m_{\pi^0}^2}{4E_\gamma}$

The minimum energetic pion to obtain the same photon energy

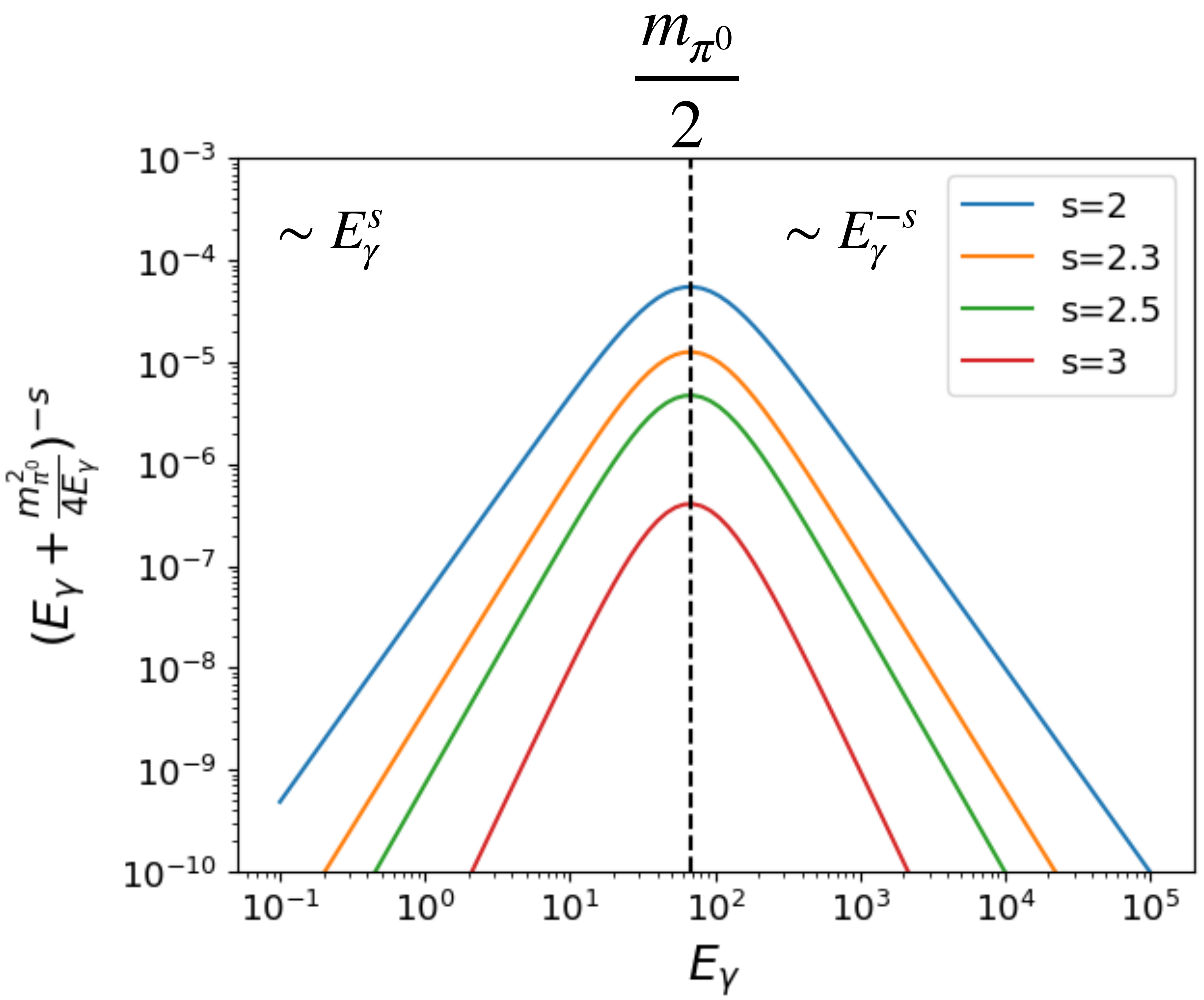
From pions to photons

The shape of photon spectrum: the pion bump

$$\frac{dN_\pi}{dE_\pi} \propto (E_\pi)^{-s} \approx (E_\pi^{min})^{-s}$$

$$E_\pi^{min} = E_\gamma + \frac{m_{\pi^0}^2}{4E_\gamma}$$

$$\begin{cases} E_\pi^{min} \sim \dfrac{m_{\pi^0}^2}{4E_\gamma} & \text{for } E_\gamma \ll \dfrac{m_{\pi^0}}{2} \quad \sim E_\gamma^s \\ E_\pi^{min} \sim E_\gamma & \text{for } E_\gamma \gg \dfrac{m_{\pi^0}}{2} \quad \sim E_\gamma^{-s} \end{cases}$$



From pions to photons

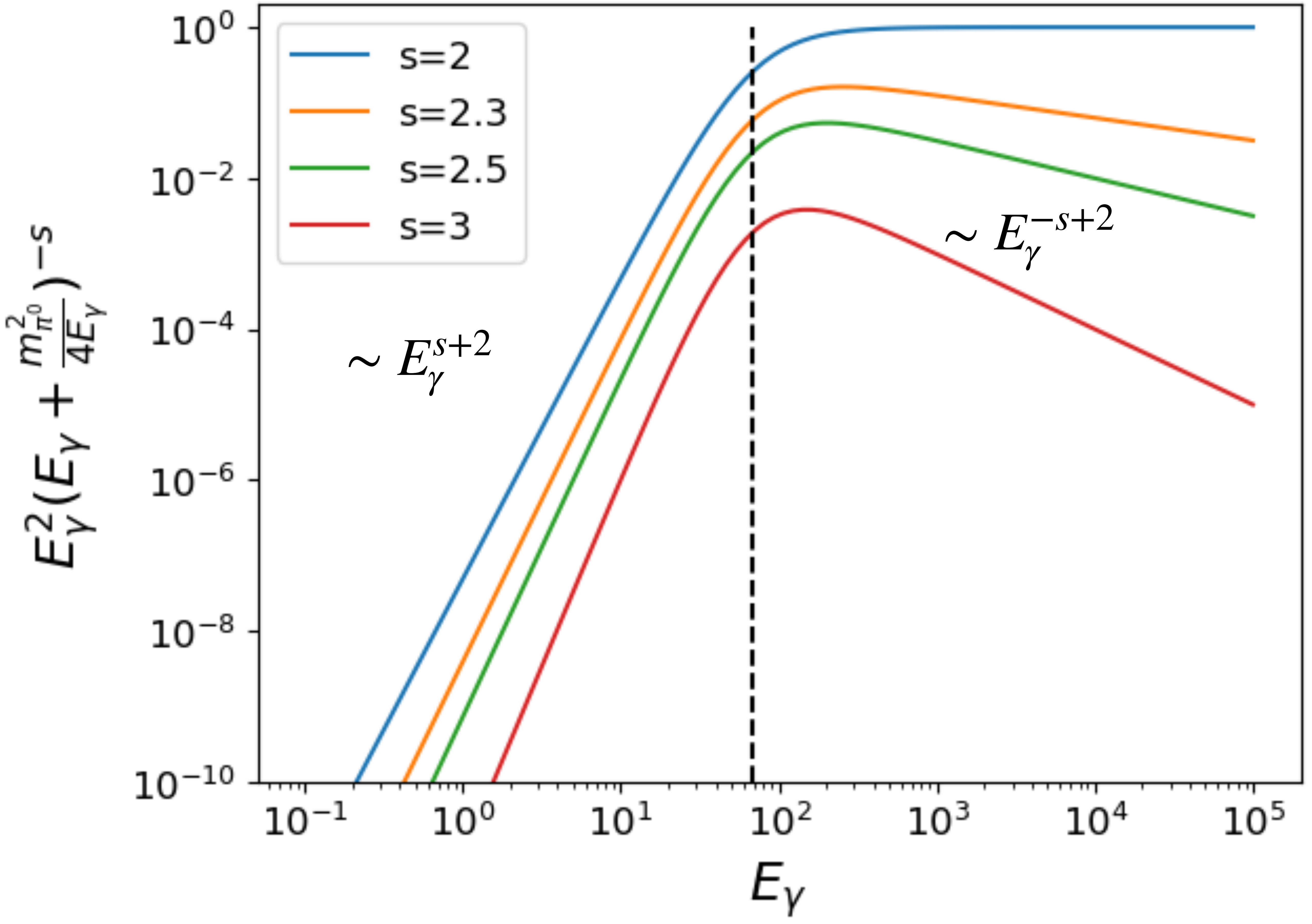
The shape of photon spectrum: the pion bump $\frac{m_{\pi^0}}{2}$

In the E^2 representation

$$\frac{dN_\pi}{dE_\pi} \propto (E_\pi)^{-s} \approx (E_\pi^{min})^{-s}$$

$$\boxed{E_\pi^{min} = E_\gamma + \frac{m_{\pi^0}^2}{4E_\gamma}}$$

$$\begin{cases} E_\pi^{min} \sim \dfrac{m_{\pi^0}^2}{4E_\gamma} & \text{for } E_\gamma \ll \dfrac{m_{\pi^0}}{2} \quad \sim E_\gamma^s \\ E_\pi^{min} \sim E_\gamma & \text{for } E_\gamma \gg \dfrac{m_{\pi^0}}{2} \quad \sim E_\gamma^{-s} \end{cases}$$

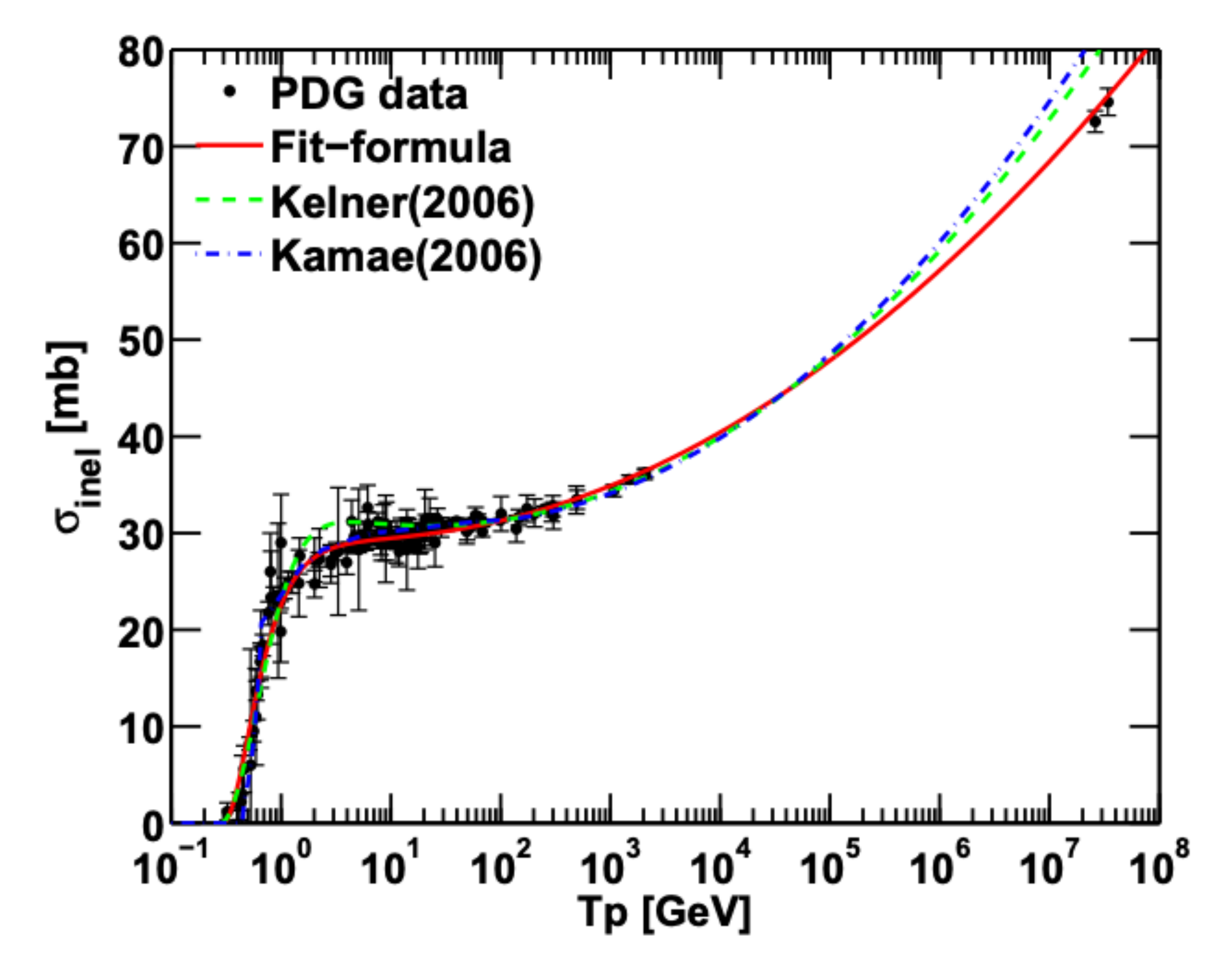


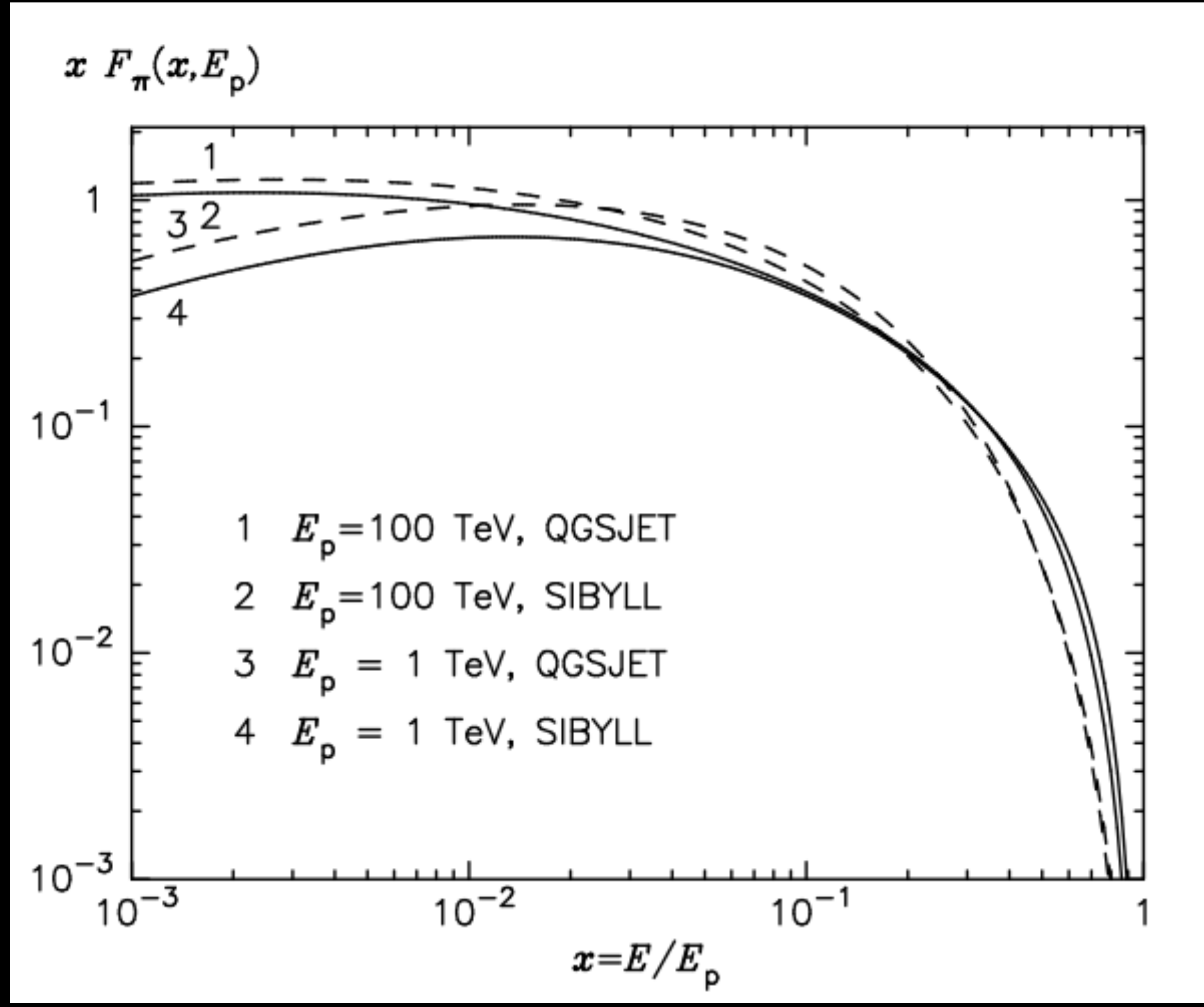
Putting everything together

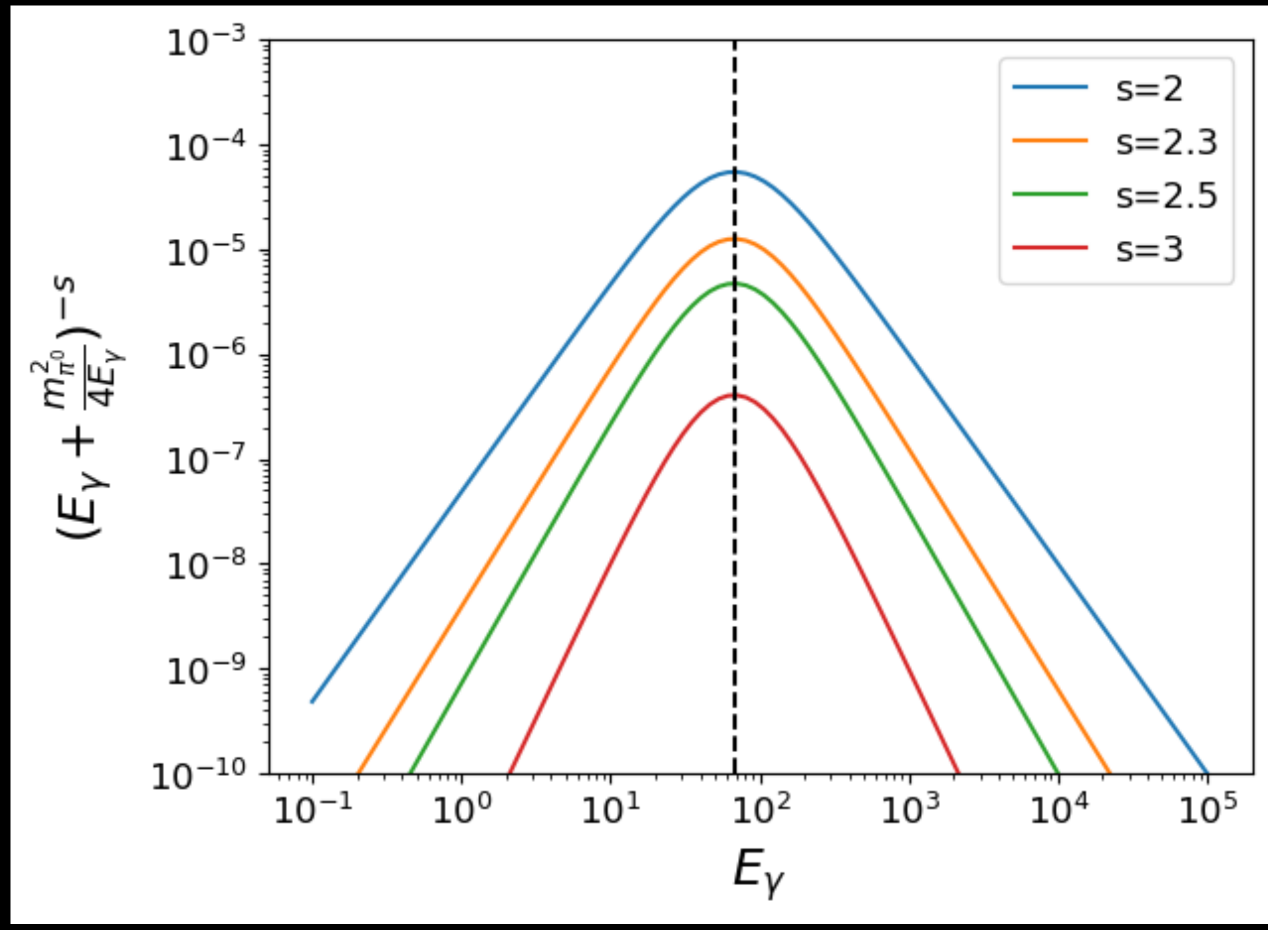
Energetic nuclei impact on ambient nuclei pp

The interaction produces unstable mesons (e.g. pions)

The pions quickly decay into gamma rays







$$\frac{dN_p}{dE_p}$$

$$\frac{d\sigma_\pi}{dE_\pi} \propto \sigma^{inel} F_\pi(x, E_p)$$

$$\frac{dN_\gamma}{dE_\gamma}$$

$$\frac{d\sigma}{dE_\gamma} = 2\int \frac{d\sigma_\pi}{dE_\pi} \frac{dE_\pi}{\sqrt{E_\pi^2 - m_{\pi^0}^2}} = 2\int \sigma_{pp}^{inel} F_\pi(x, E_p) \frac{dE_\pi}{\sqrt{E_\pi^2 - m_{\pi^0}^2}}$$

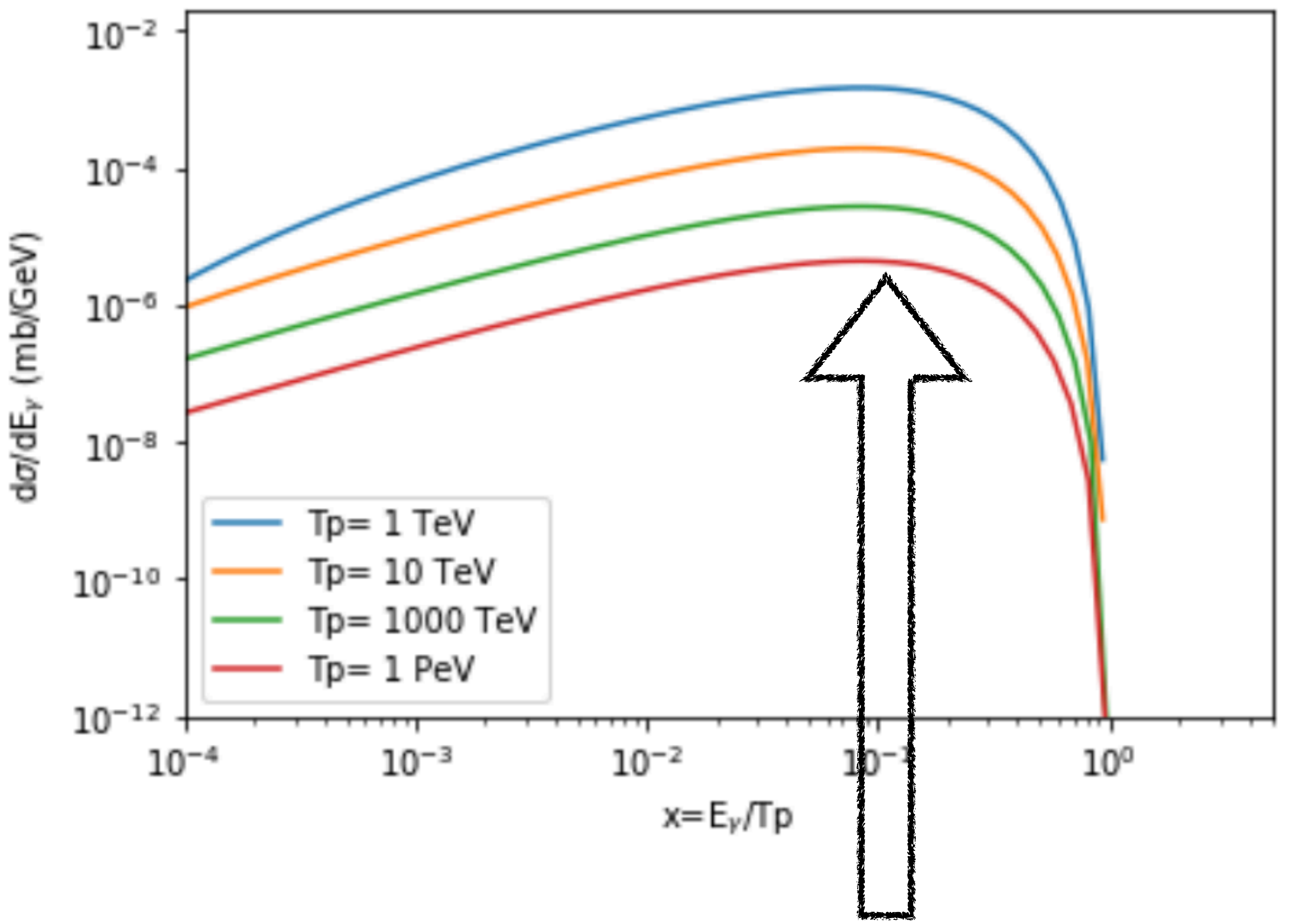
Putting everything together

Connecting protons and gamma

$$\frac{d\sigma}{dE_\gamma} = 2\int \frac{d\sigma_\pi}{dE_\pi}\frac{dE_\pi}{\sqrt{E_\pi^2 - m_{\pi^0}^2}} = 2\int \sigma_{pp}^{inel} F_\pi(x, E_p)\frac{dE_\pi}{\sqrt{E_\pi^2 - m_{\pi^0}^2}}$$

$$\frac{dN_\gamma}{dE_\gamma} = \ldots = cn_H \int dE_p \frac{d\sigma_{pp\to 2\gamma}}{dE_\gamma}\frac{dN_p}{dE_p}$$

Contains all info about pion production and multiplicity
Derived with numerical methods



Photon get ~10% of proton energy

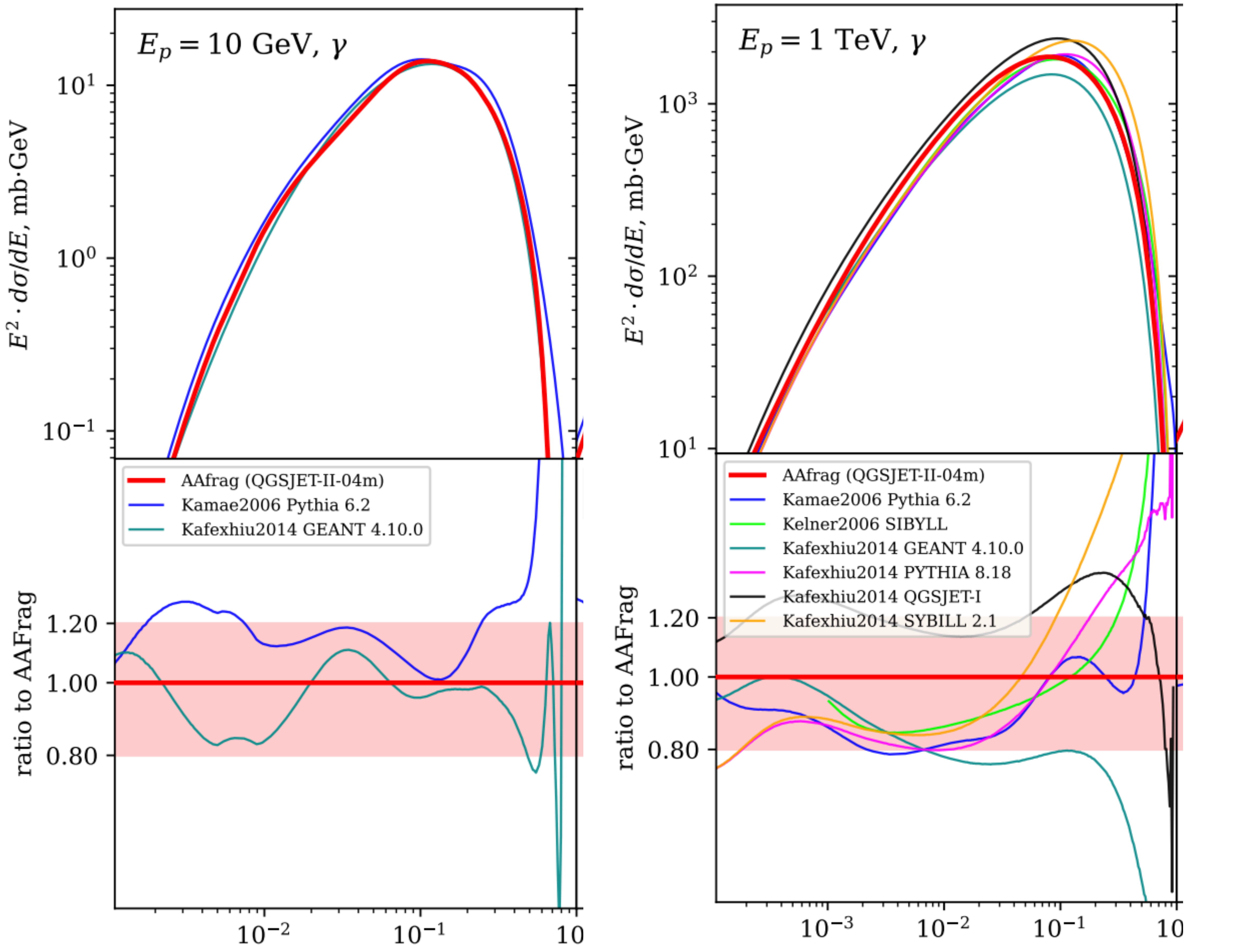
Putting everything together

Connecting protons and gamma

$$\frac{dN_\gamma}{dE_\gamma} = \ldots = c n_H \int dE_p \frac{d\sigma_{pp\to 2\gamma}}{dE_\gamma} \frac{dN_p}{dE_p}$$

Contains all info about pion production and multiplicity
Derived with numerical methods

Systematic difference due to different parametrization

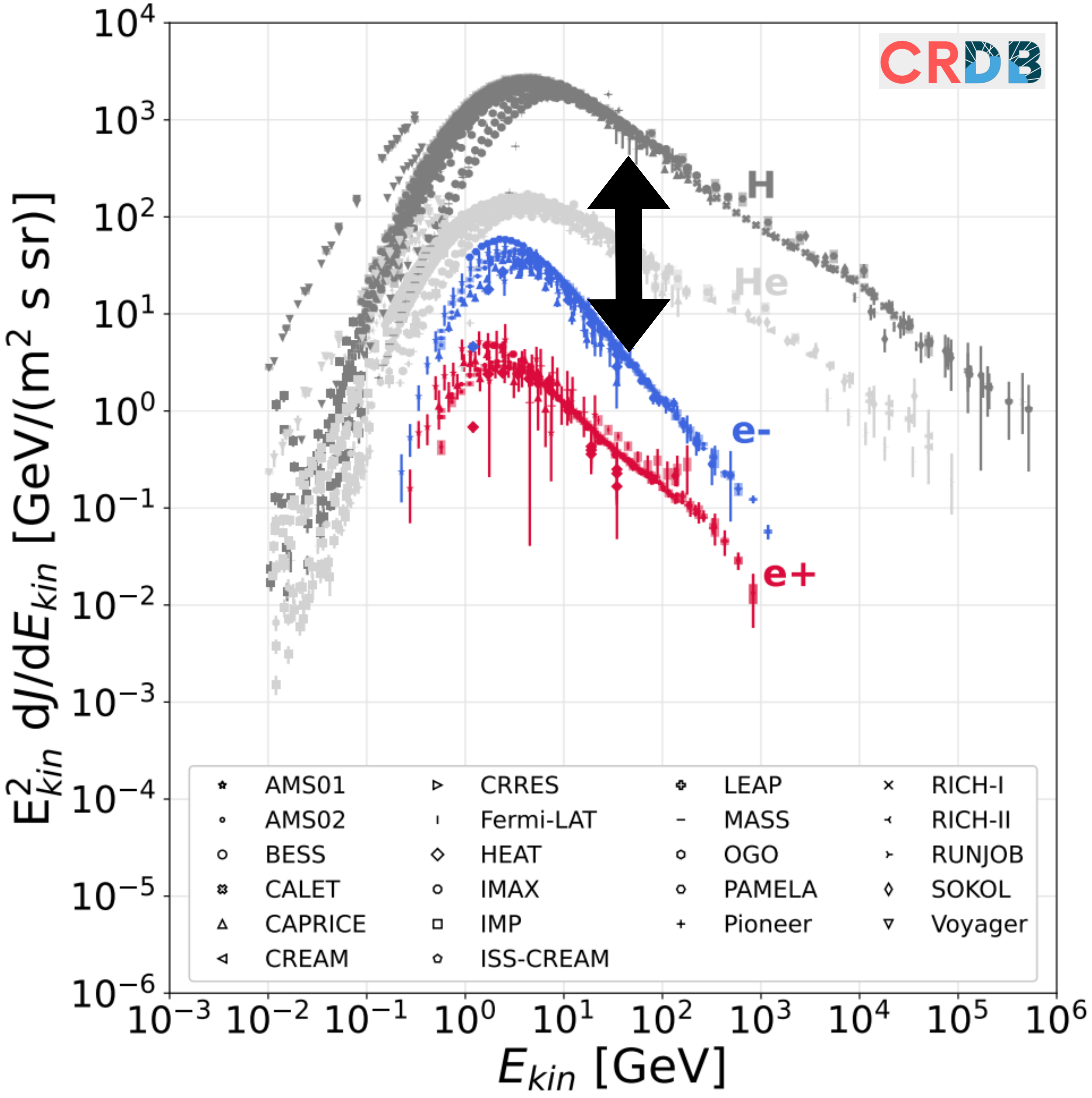


X = E/Tp

Hadronic interactions

And the origin of Cosmic Rays

Hadrons ~ 99%
Leptons ~ 1%

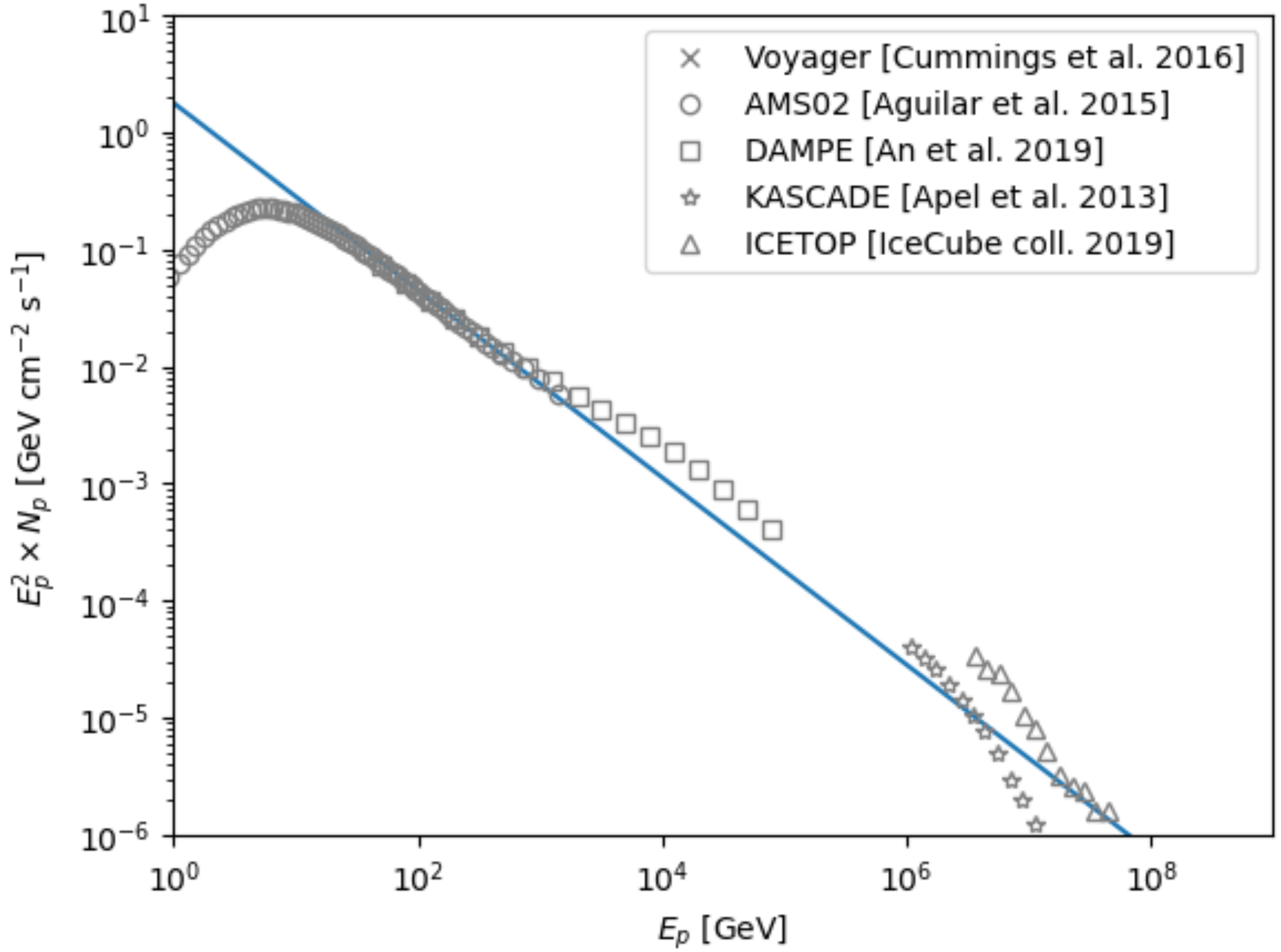


Protons dominate the CR composition: *understanding CRs is tightly linked to hadronic interactions*

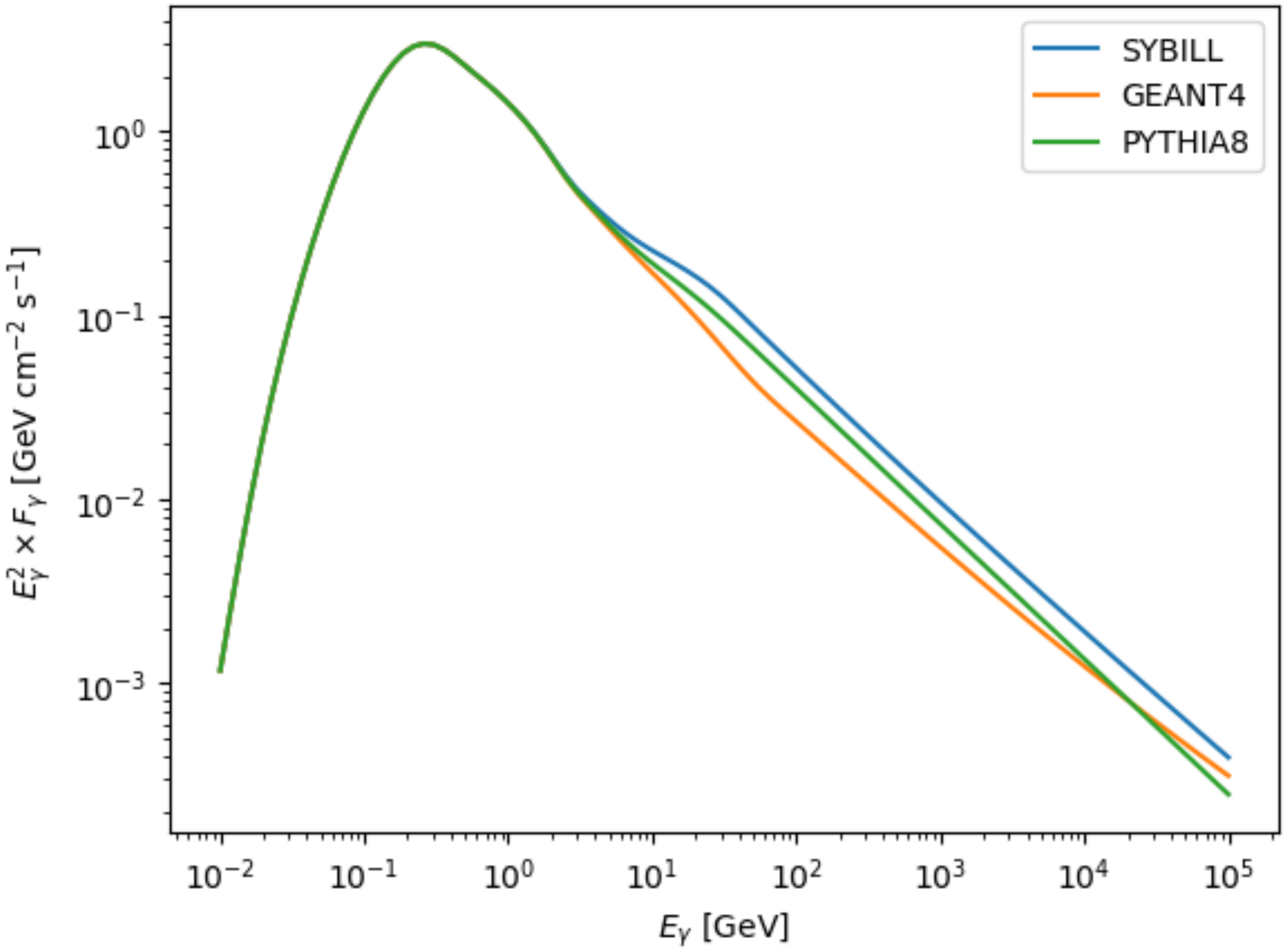
An example

The emissivity of the cosmic-ray sea

Protons

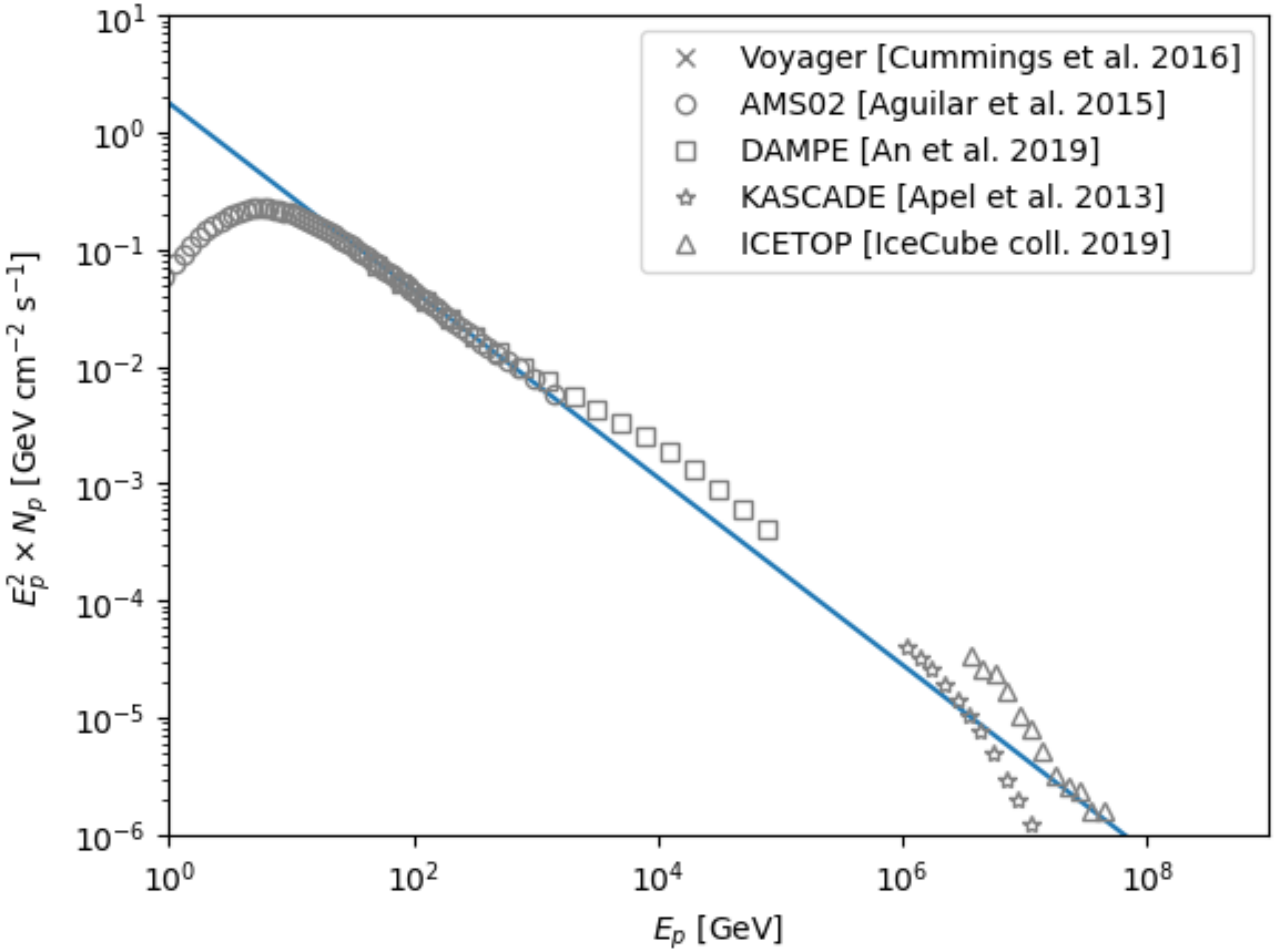


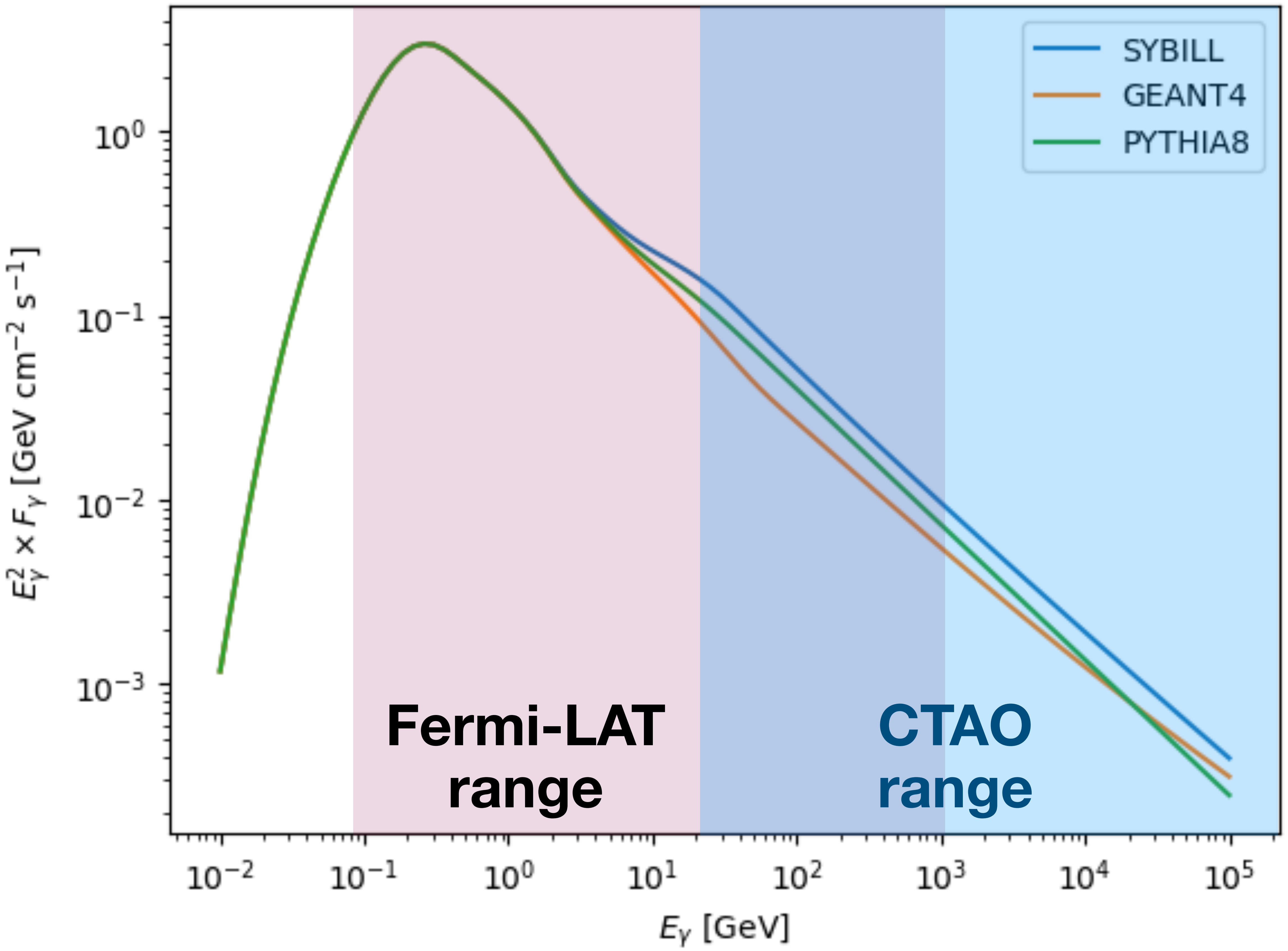
Gamma rays



An example

The emissivity of the cosmic-ray sea

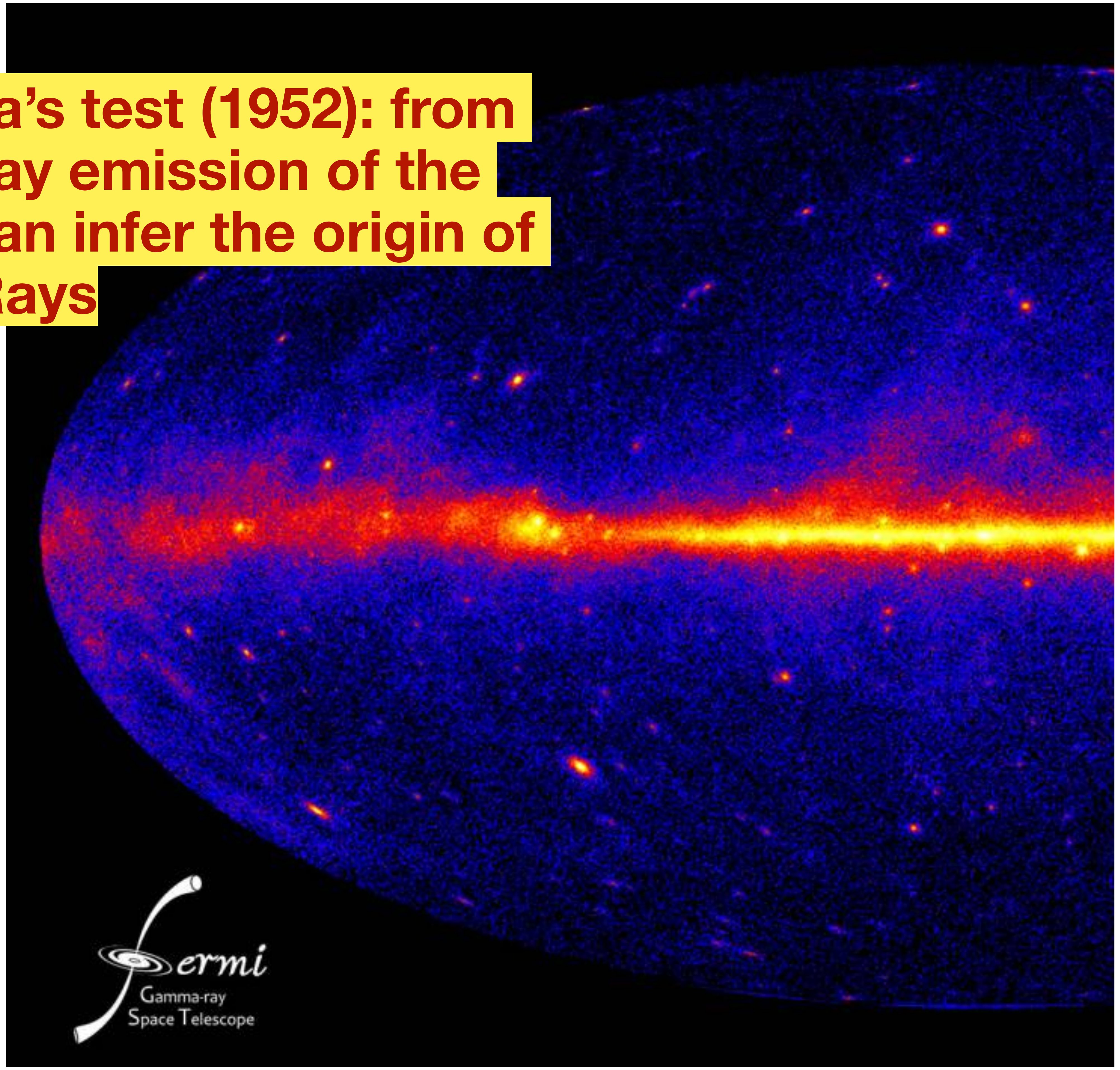


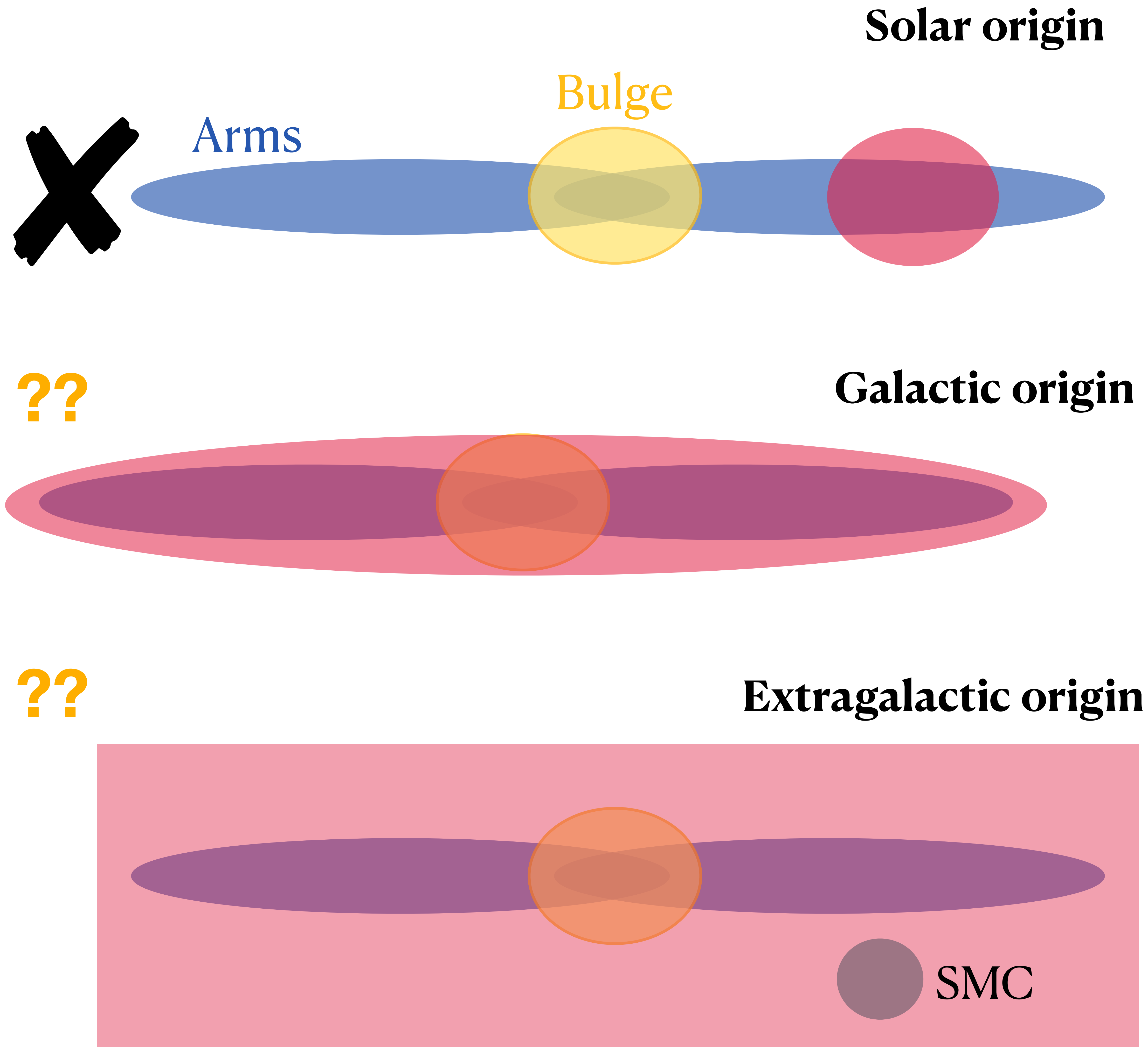


Hadronic interactions

And the origin of Cosmic Rays

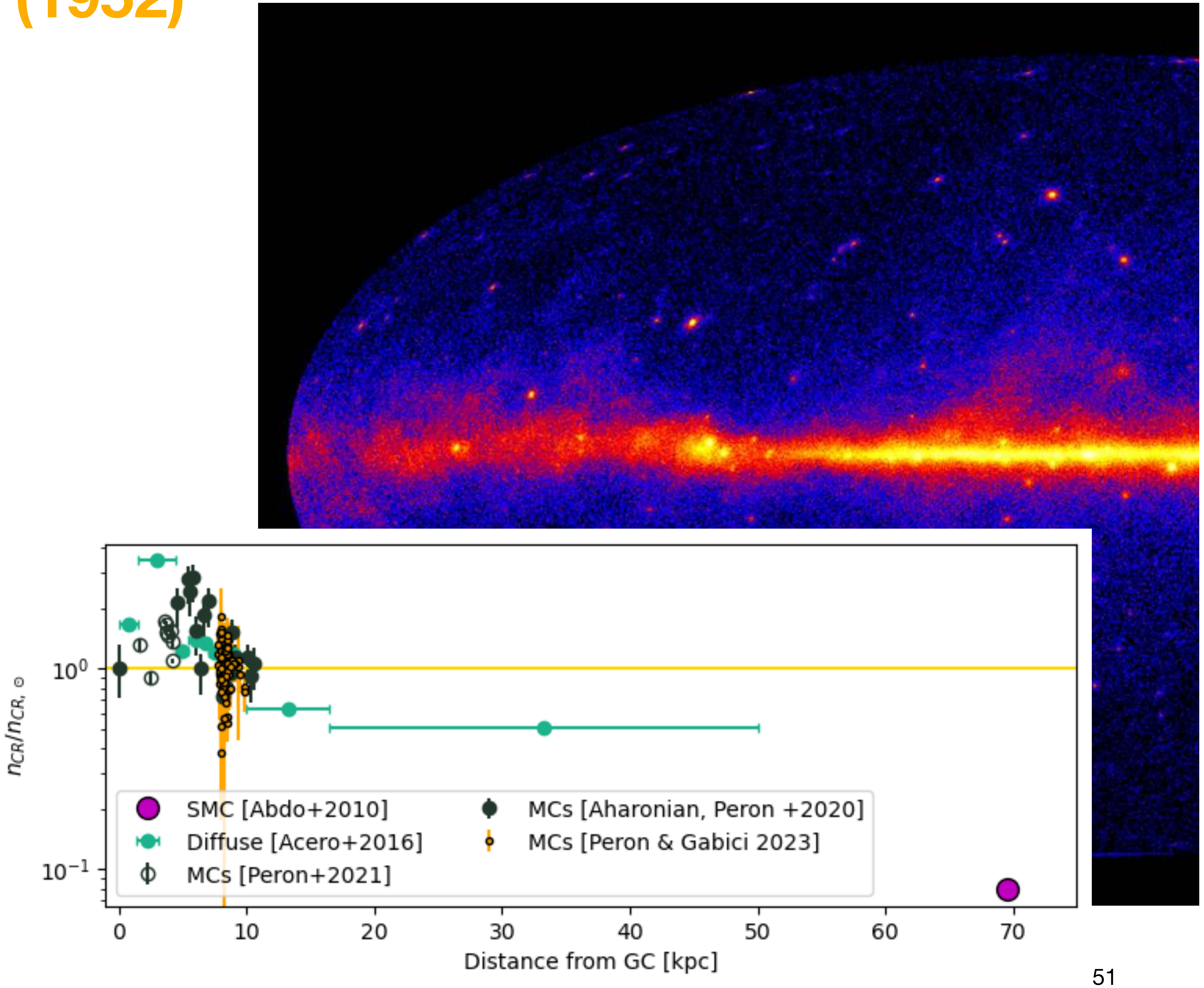
Hayakawa's test (1952): from gamma-ray emission of the ISM we can infer the origin of Cosmic Rays



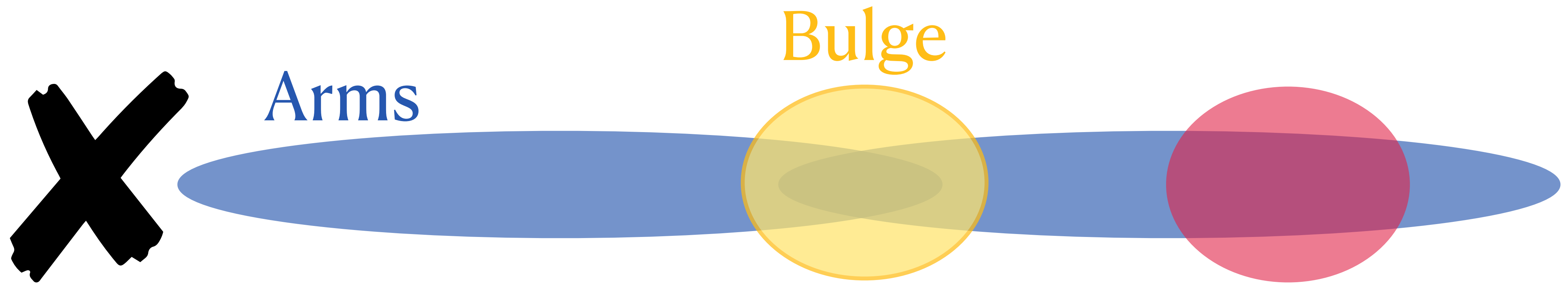


Hadronic interactions

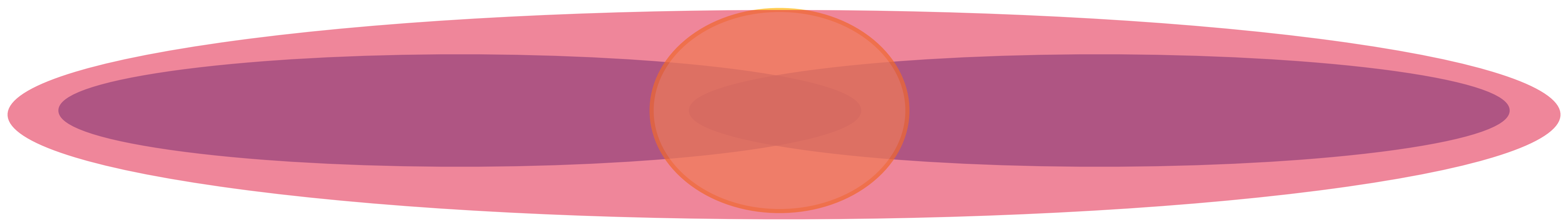
Hayakawa's test: from gamma-ray emission of the ISM we can infer the origin of Cosmic Rays (1952)



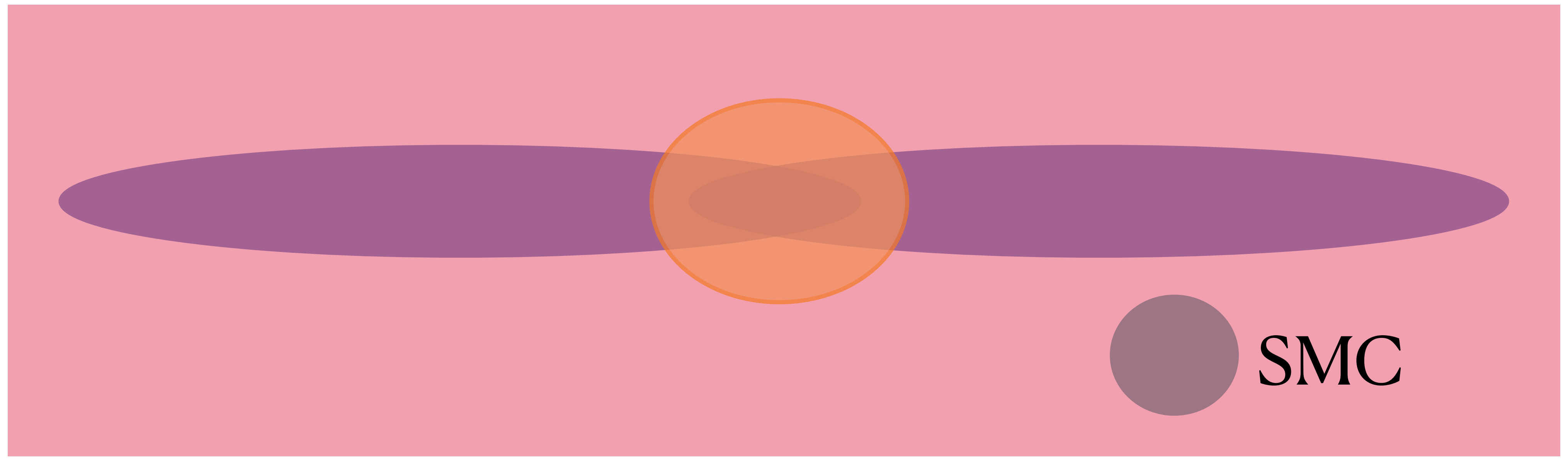
Solar origin



Galactic origin



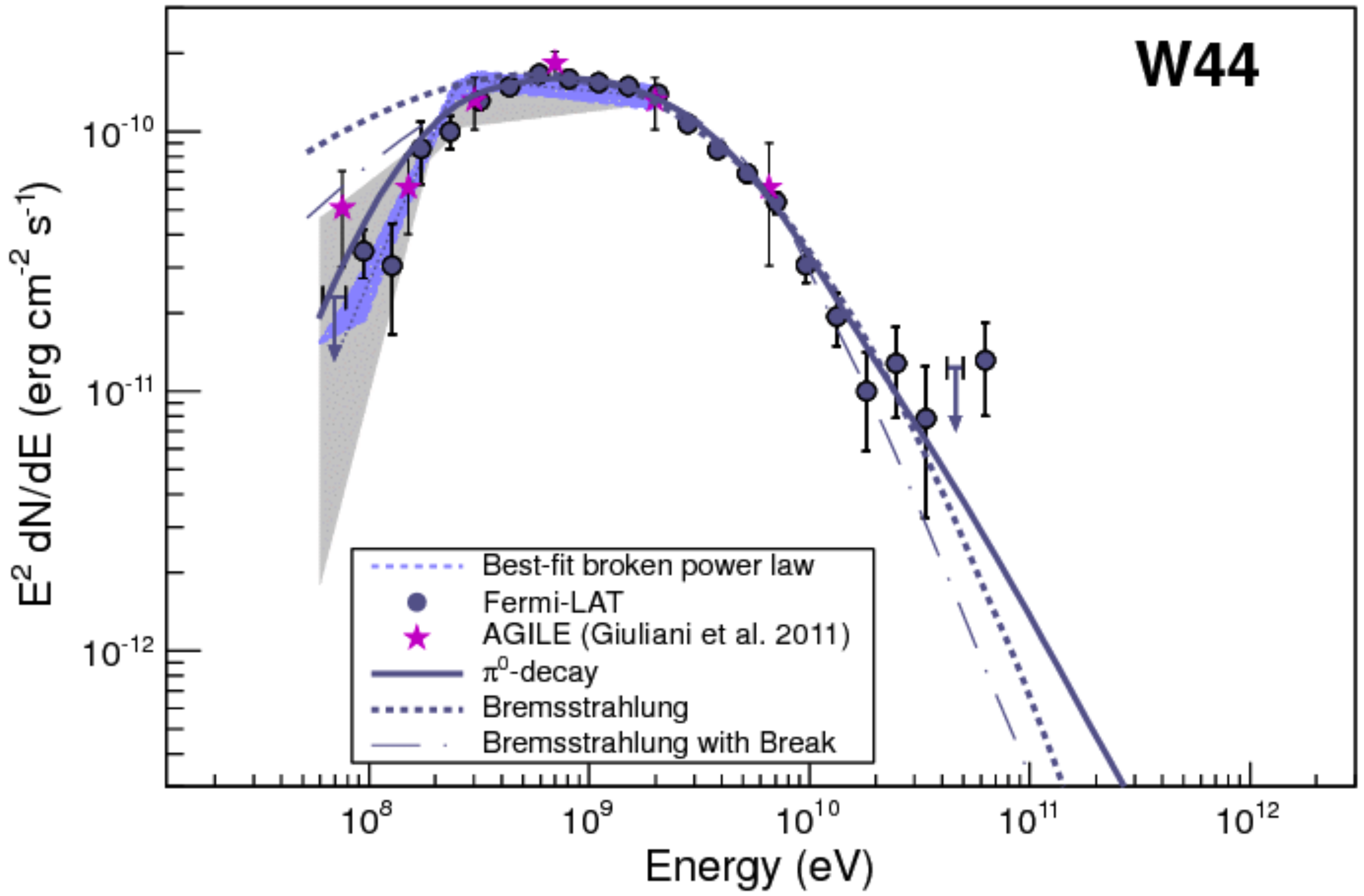
Extragalactic origin

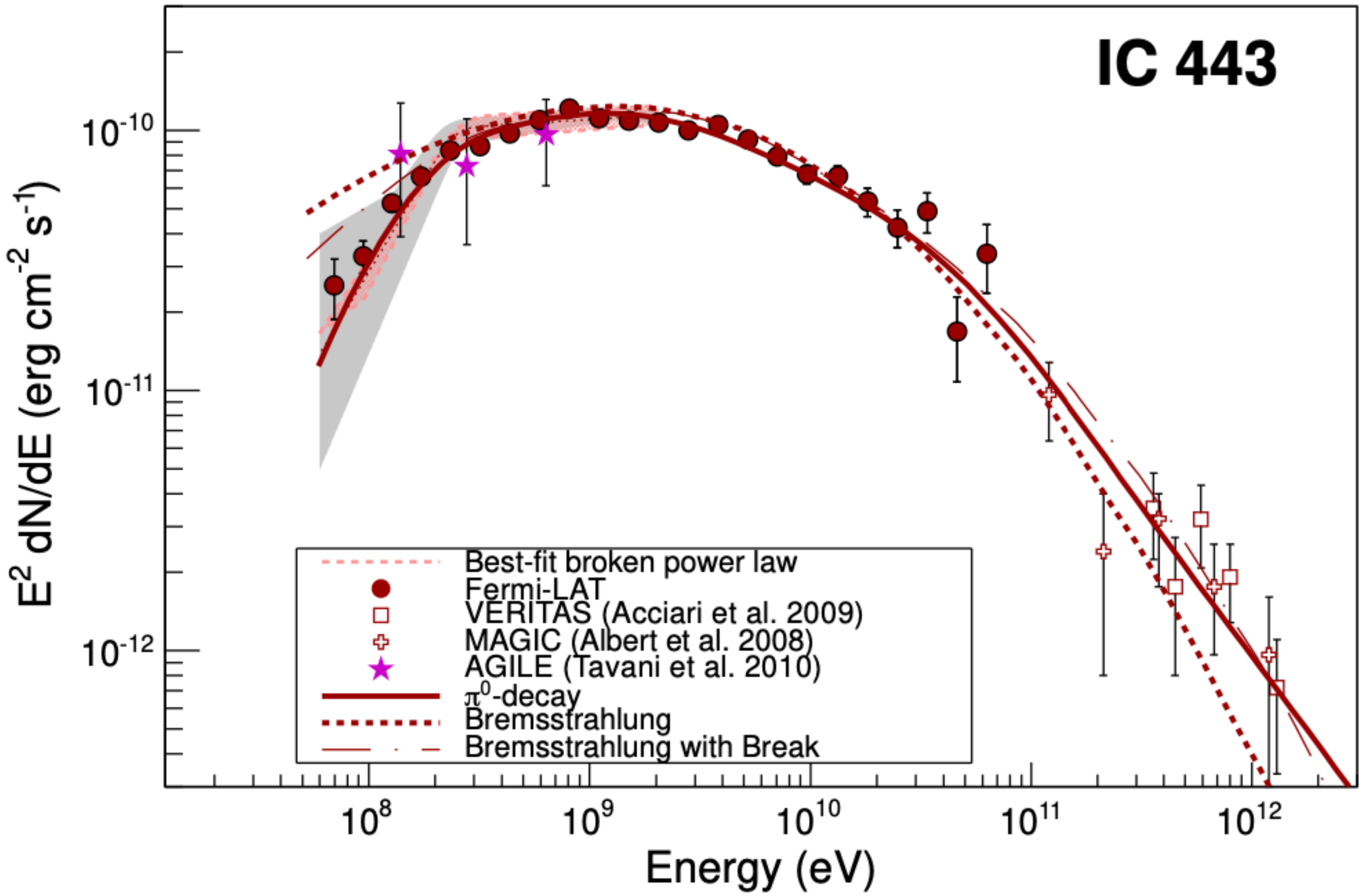


The pion bump in SNRs

Sources of cosmic rays in the Galaxy

Pion bump detected in SNRs supports the fact that SNRs are accelerating protons of CRs



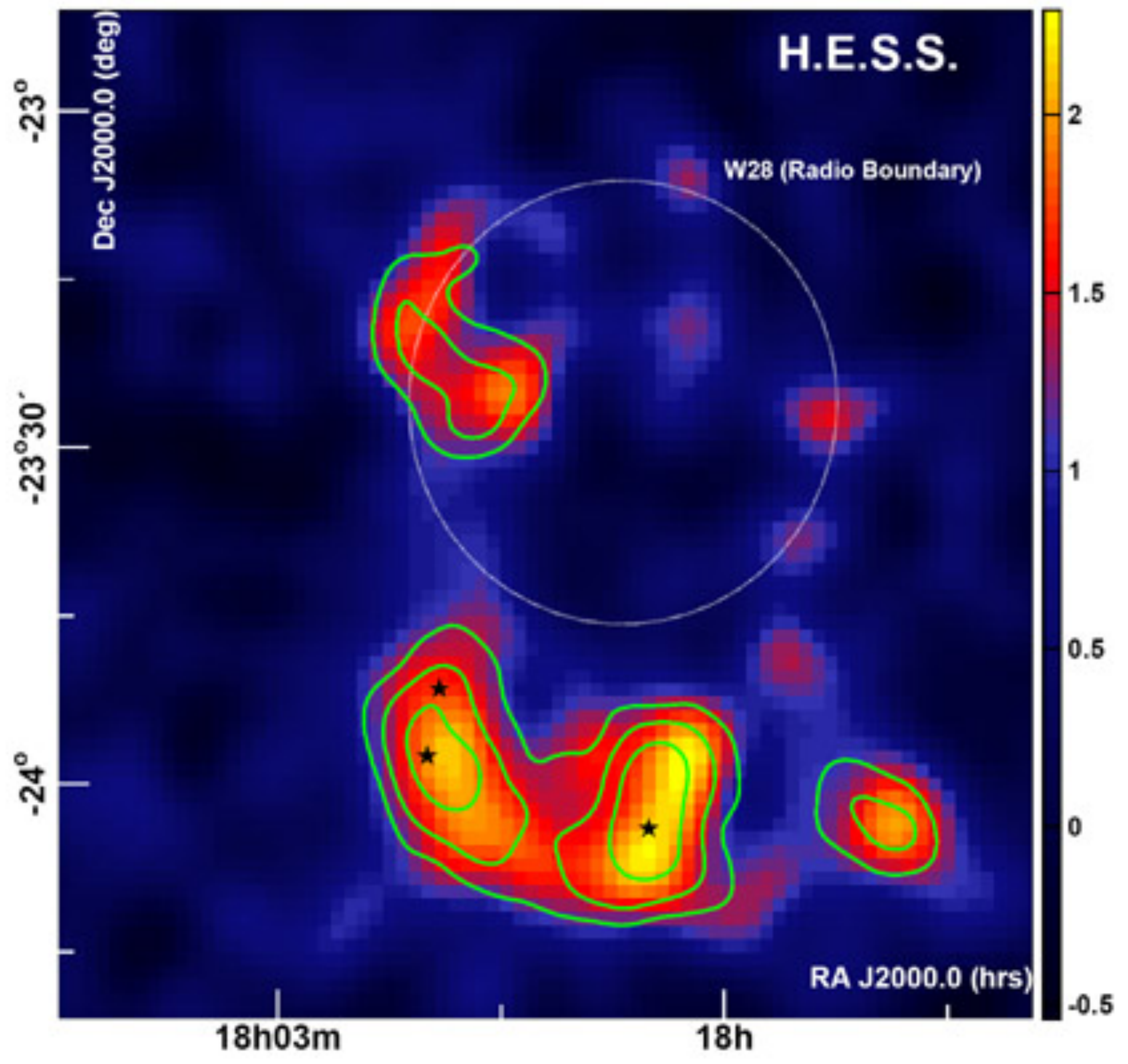


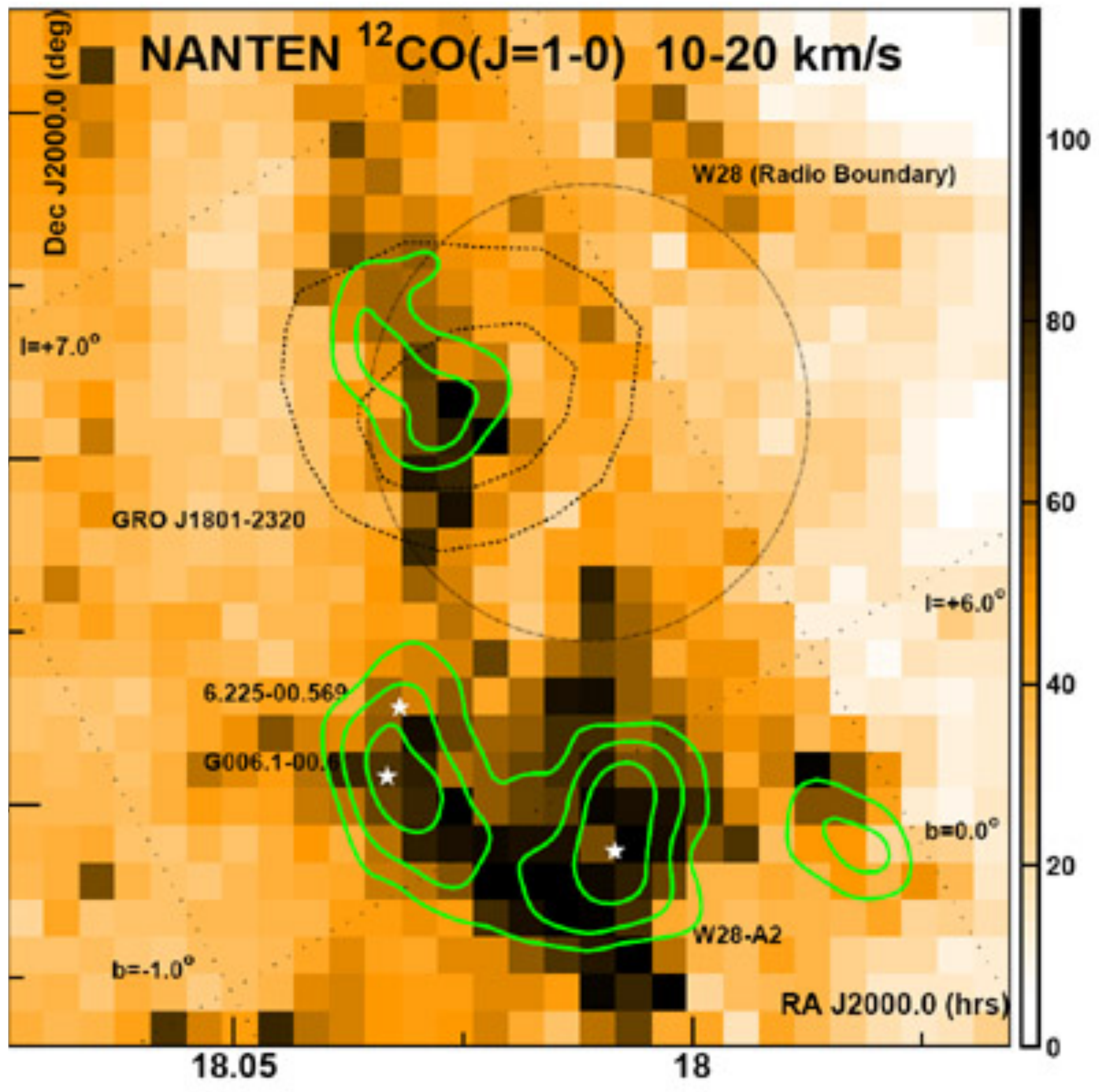
..not always visible, and need low energy observations: we need to search for other features (correlation with gas structure or **multi-wavelength data to constrain the leptonic counterpart —> we will actively do it in the mini-project!**)

Hadronic emission

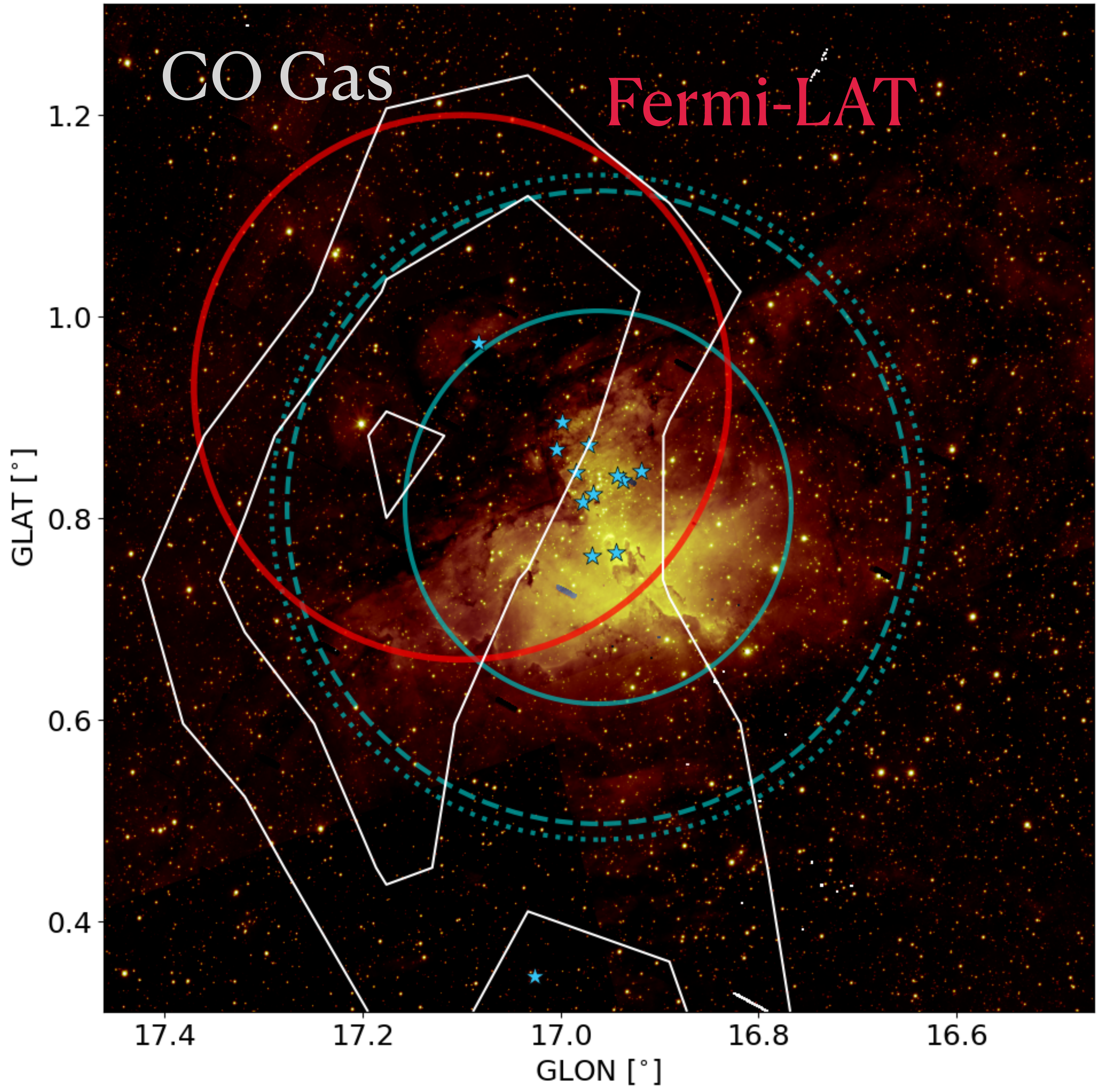
Gamma vs. gas

Middle age SNR W28



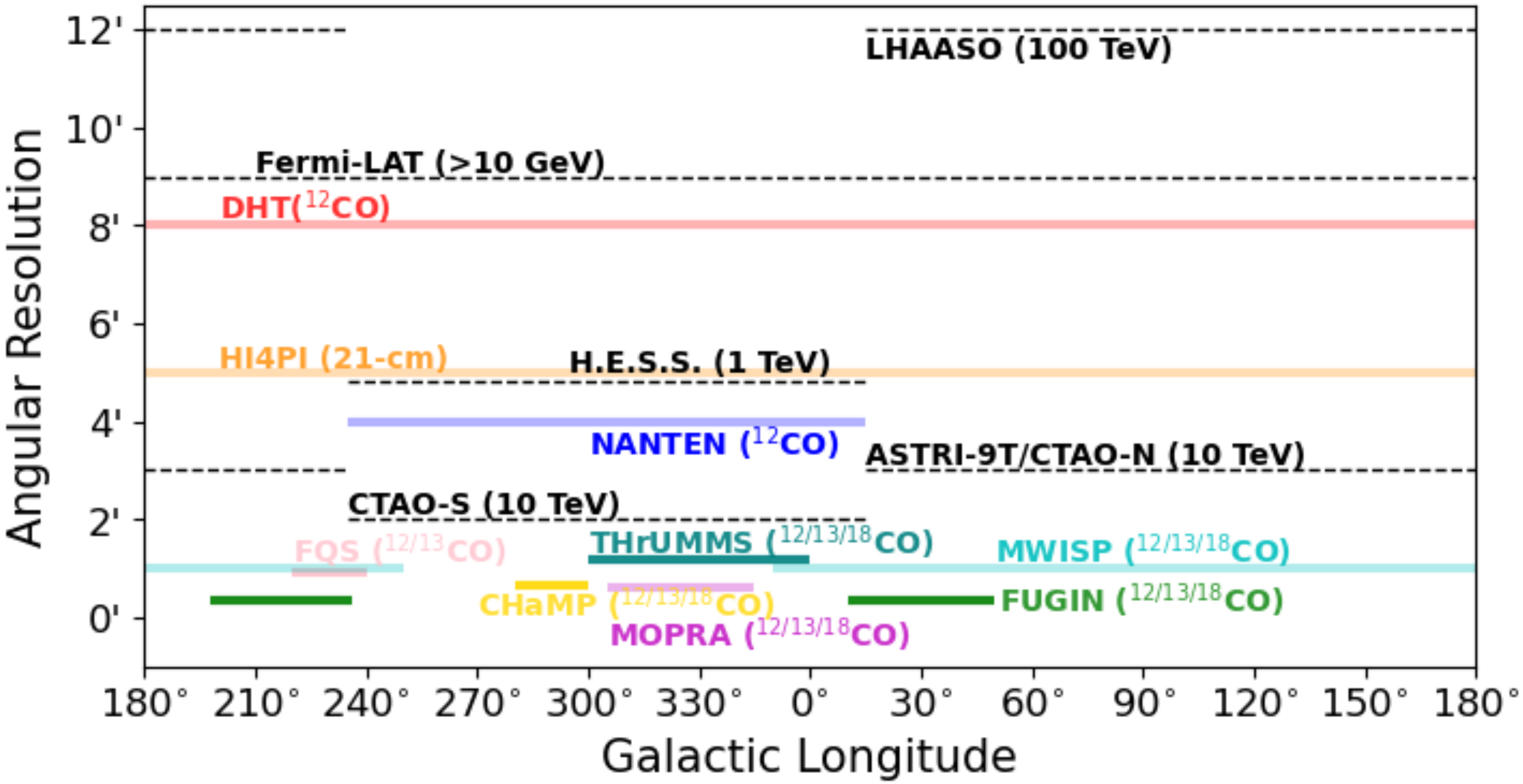


Young star cluster NGC661



Hadronic emission

Gamma vs. Gas

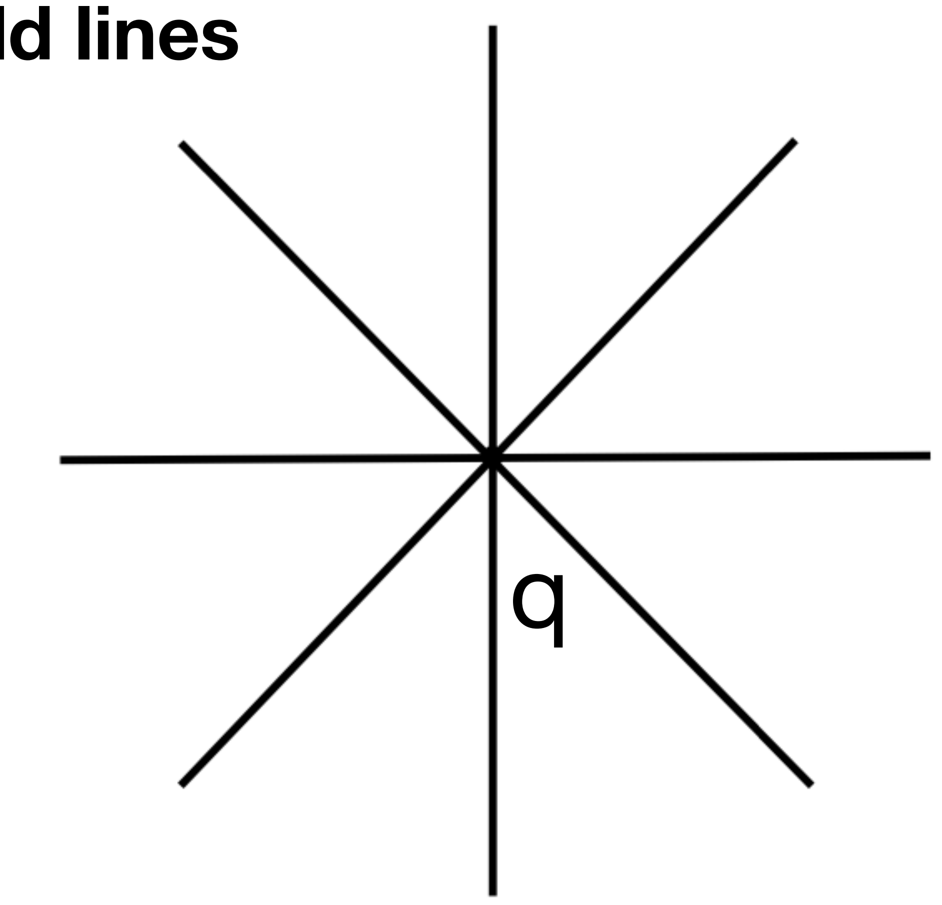


Leptonic Interactions

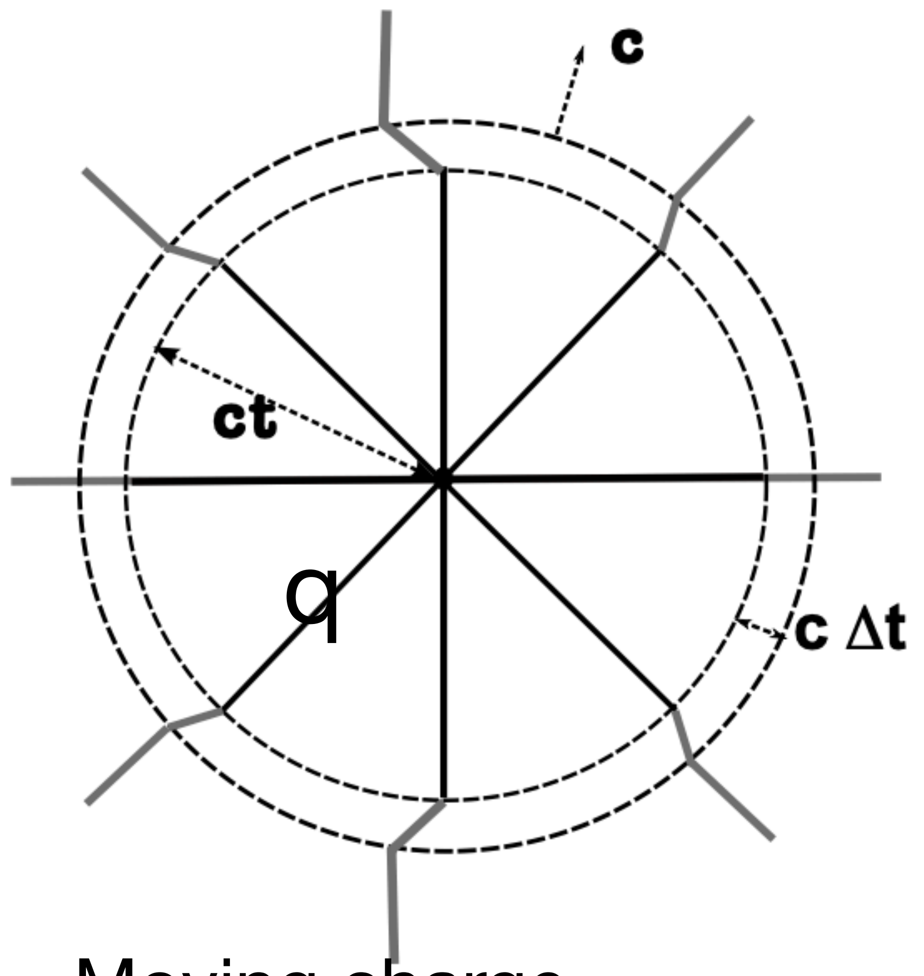
Radiation from a moving charge

Larmor formula: Heuristic, non-relativistic derivation

E-field lines



Still charge



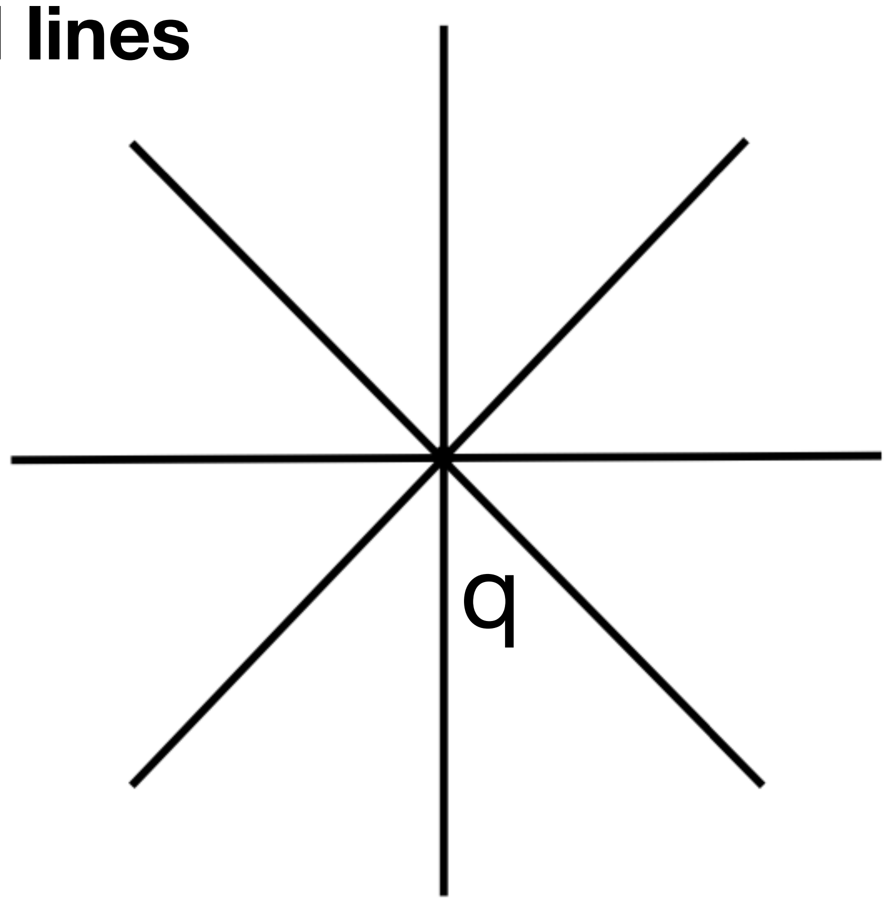
Moving charge

Field outside r=ct is still pointing toward the center at t-Δt

Radiation from a moving charge

Larmor formula: Heuristic, non-relativistic derivation

E-field lines



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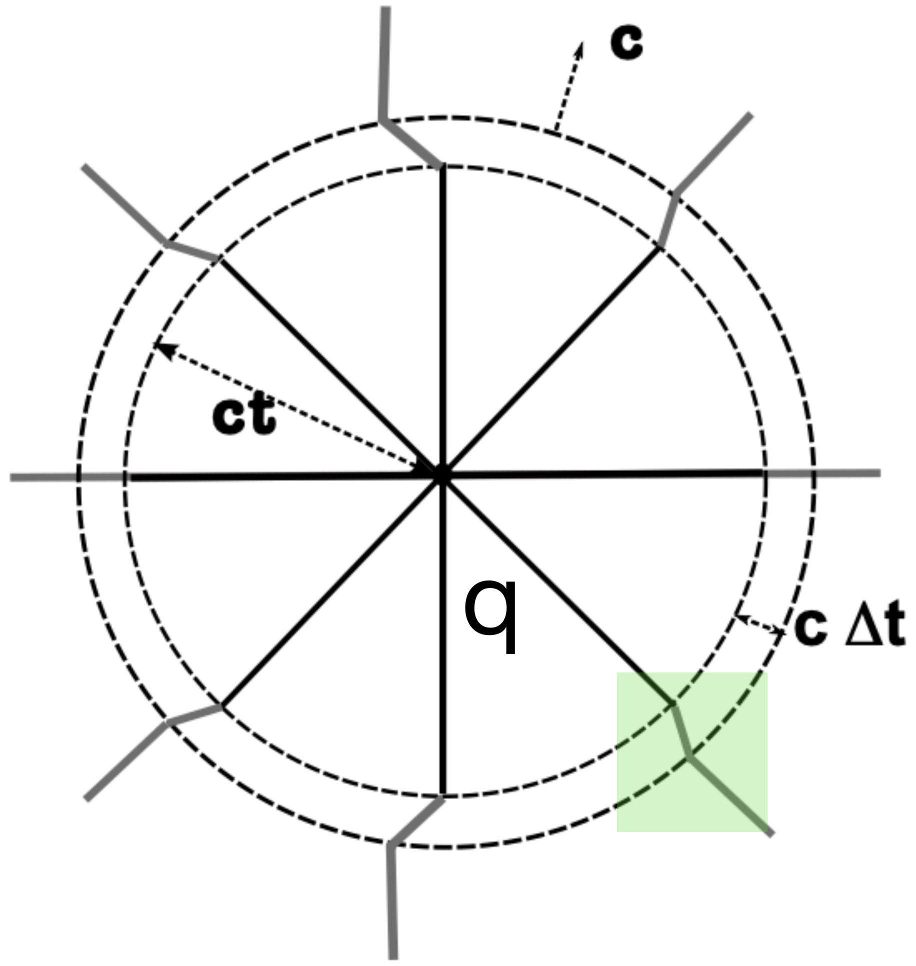
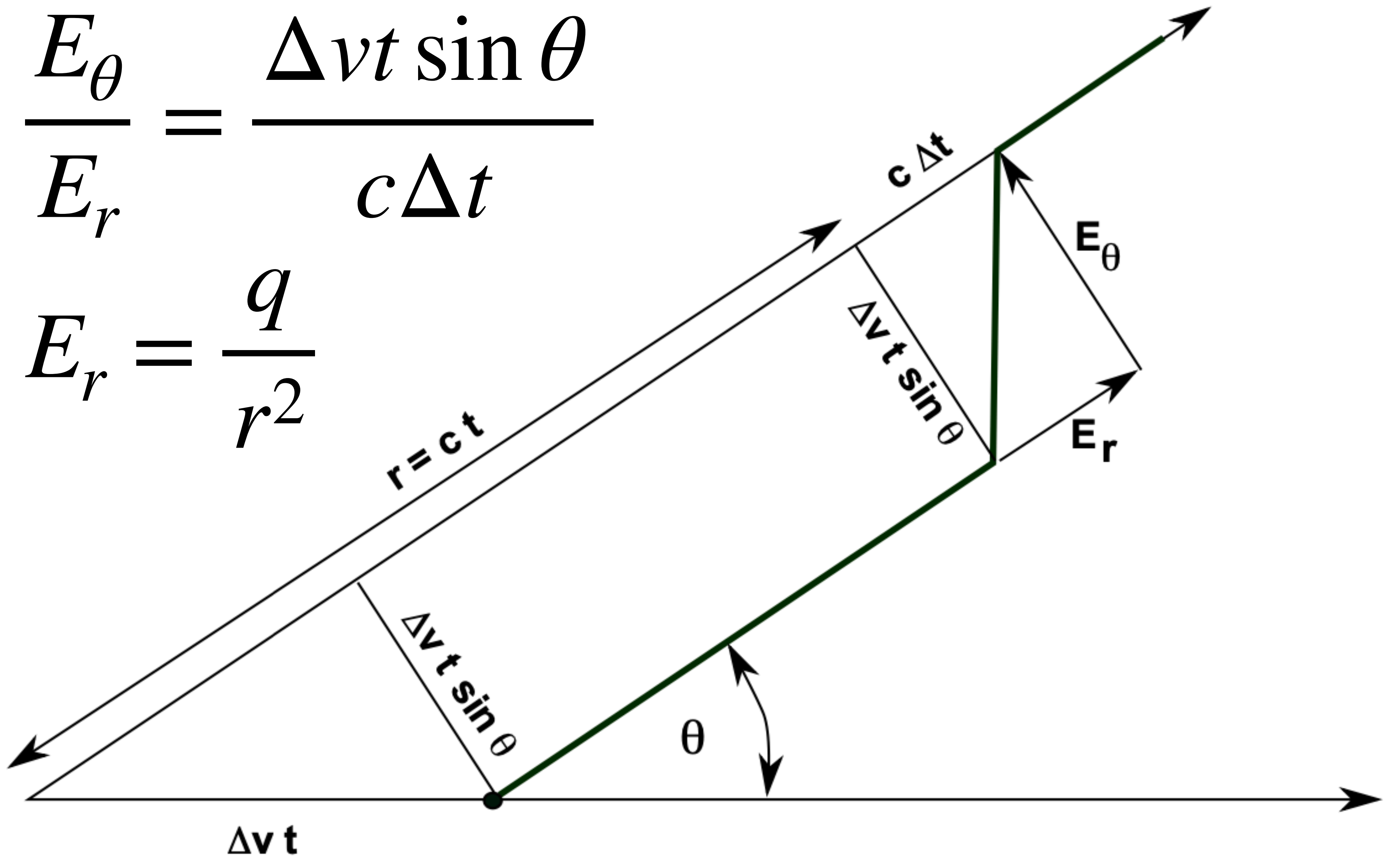
Field outside r=ct is still pointing toward the center at t-Δt



Moving charge

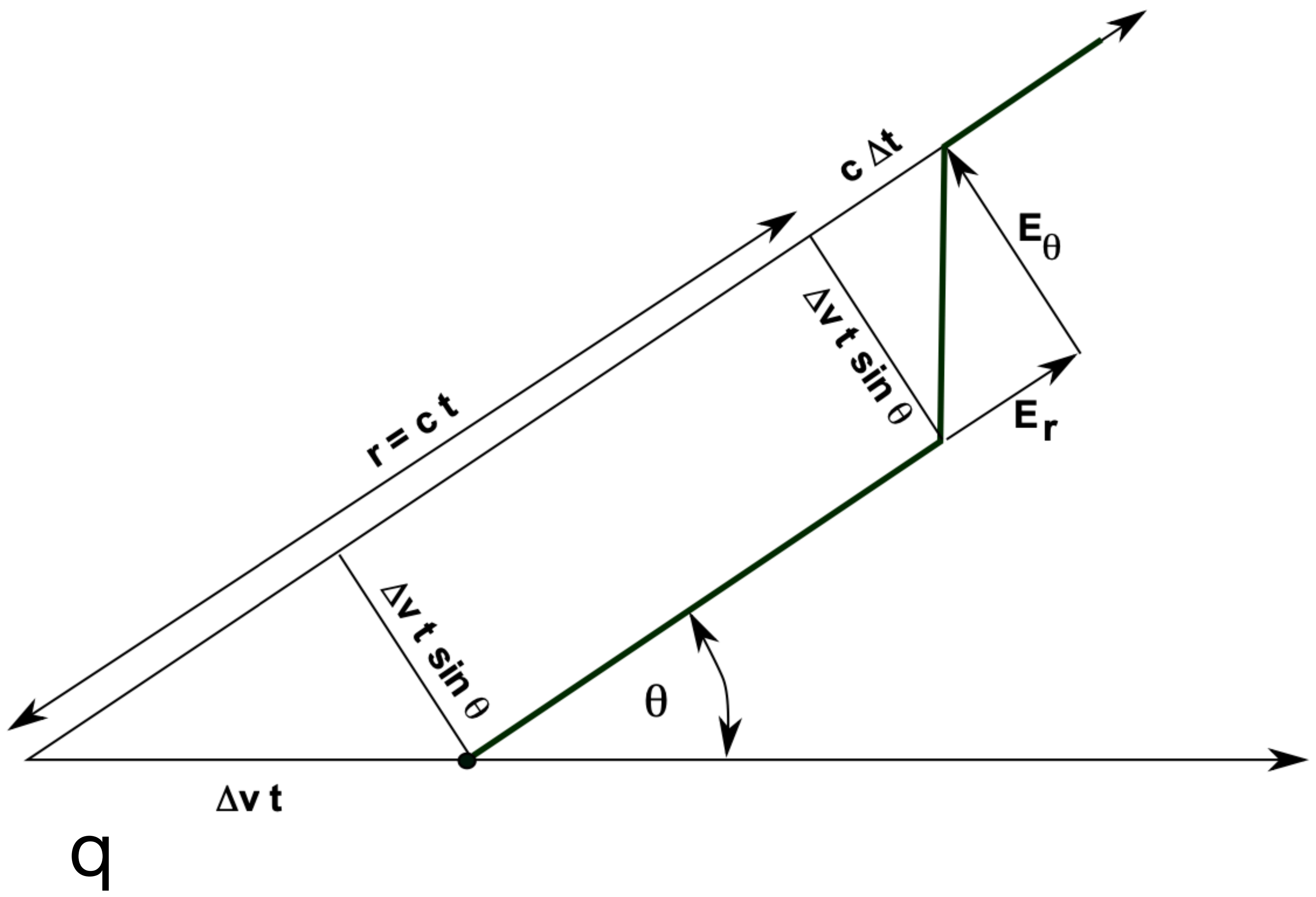
$$\frac{E_\theta}{E_r} = \frac{\Delta v t \sin\theta}{c\Delta t}$$

$$E_r = \frac{q}{r^2}$$



Radiation from a moving charge

Larmor formula: Heuristic, non-relativistic derivation



$$\left.\begin{aligned} \frac{E_\theta}{E_r} &= \frac{\Delta v t \sin\theta}{c\Delta t} \\ E_r &= \frac{q}{r^2} \quad \text{Gauss law [cgs]} \end{aligned}\right\} \quad E_\theta = \frac{cq\Delta vt\sin\theta}{cr^2 c\Delta t} = \frac{q}{r}\underbrace{\frac{\Delta v}{\Delta t}}_{\text{Acceleration}}\frac{\sin\theta}{c^2}$$

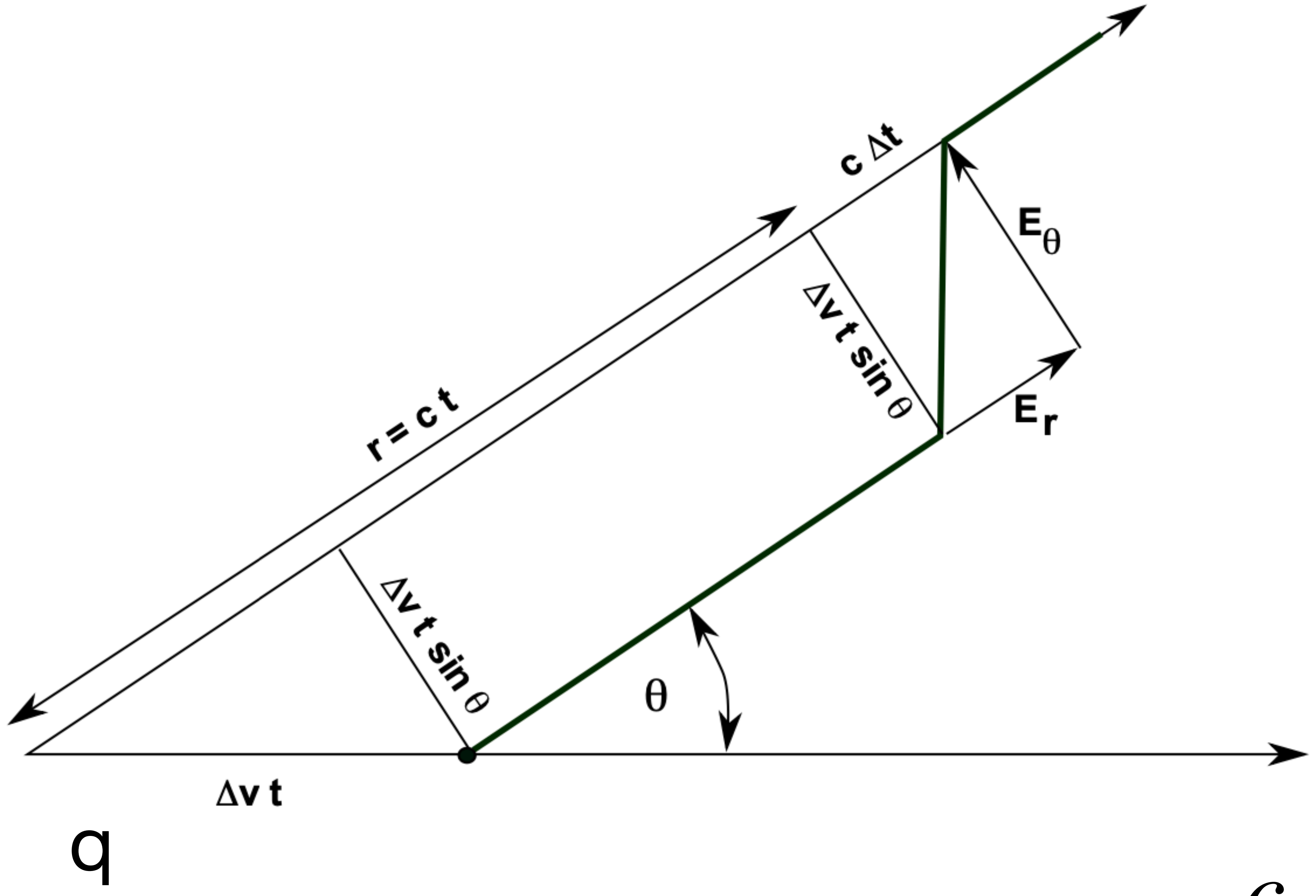
$$E_\theta = \frac{q}{rc^2}|\vec{\dot{v}}|\sin\theta$$

$$E_\theta \propto \frac{1}{r}$$

There is a non-vanishing *tangential* component of the E field that allows the flow of energy —> let's see the Poynting flux

Radiation from a moving charge

Larmor formula: Heuristic, non-relativistic derivation



$$\left.\begin{aligned}\frac{E_\theta}{E_r} &= \frac{\Delta v t \sin\theta}{c\Delta t} \\ E_r &= \frac{q}{r^2} \quad \text{Gauss law [cgs]}\end{aligned}\right\}$$

$$r = ct \qquad \text{Acceleration}$$

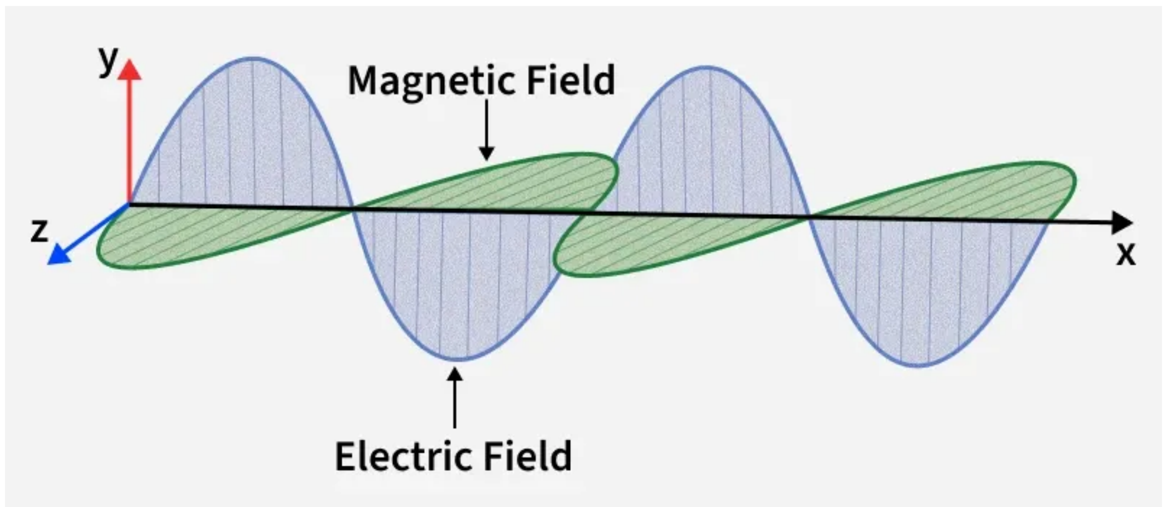
$$E_\theta = \frac{c}{c}\,\frac{q\Delta v t \sin\theta}{r^2 c\Delta t} = \frac{q\ \Delta v \sin\theta}{r\ \Delta t\ c^2}$$

$$E_\theta = \frac{q}{rc^2}|\vec{\dot{v}}|\sin\theta$$

$$\vec{n} = \frac{\vec{r}}{|r|}$$

Energy emitted per unit time and area

$$\vec{S} = \frac{c}{4\pi}((\vec{E_\theta} + \vec{E_r}) \times \vec{B}) = \frac{c}{4\pi}(E_\theta{}^2\vec{n})$$

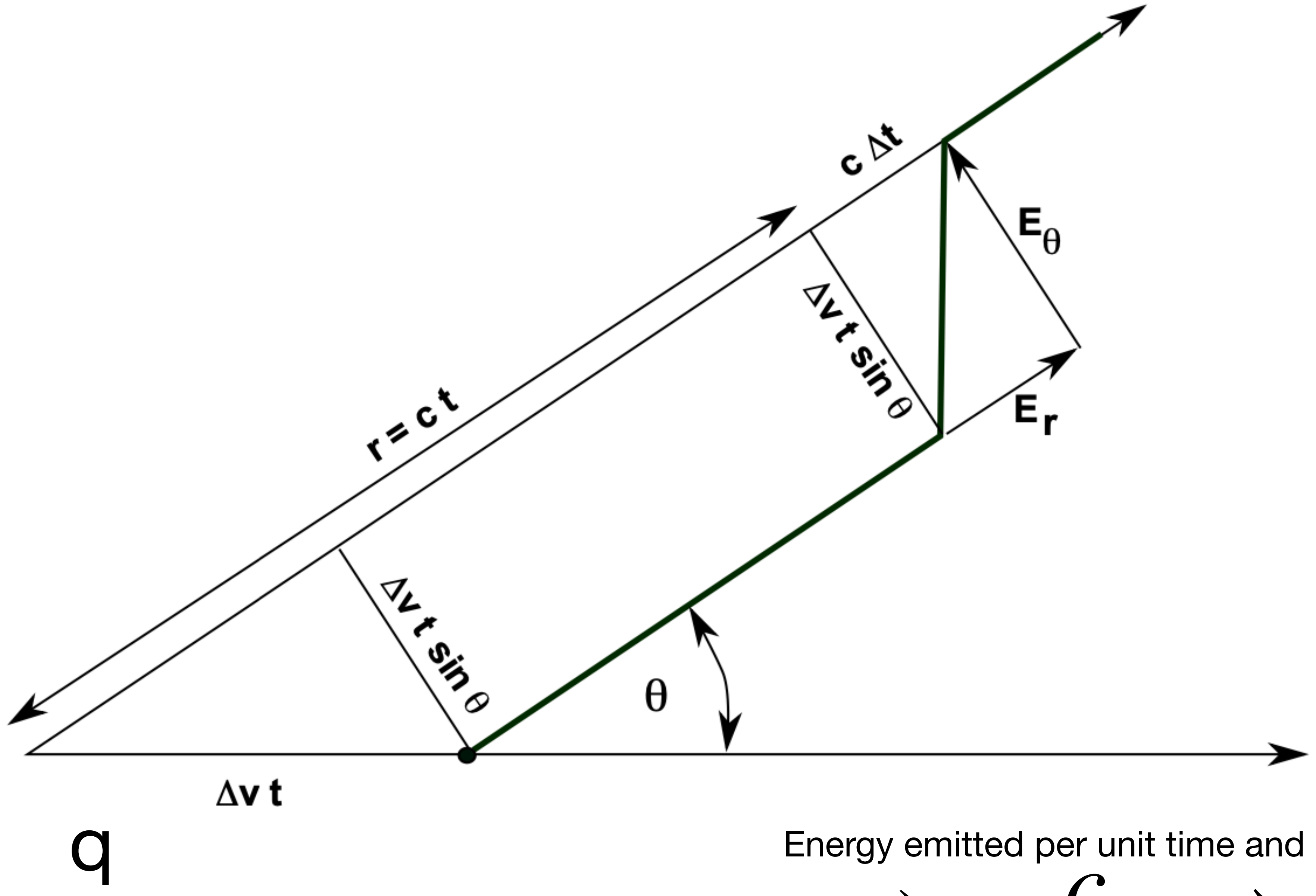


$E_\theta = B$ for a plane wave in vacuum

$\vec{S} \parallel \vec{n}$ *Outgoing propagation of energy!*

Radiation from a moving charge

Larmor formula: Heuristic, non-relativistic derivation



$$\left.\begin{aligned}\frac{E_\theta}{E_r} &= \frac{\Delta v t \sin\theta}{c\Delta t} \\ E_r &= \frac{q}{r^2} \quad \text{Gauss law [cgs]}\end{aligned}\right\} E_\theta = \frac{cq\Delta v t \sin\theta}{cr^2 c\Delta t} = \frac{q\ \Delta v \sin\theta}{r\ \Delta t\ c^2}$$

Acceleration

$$E_\theta = \frac{q}{rc^2}|\dot{\vec{v}}|\sin\theta$$

Let's compute the power :

$$\vec{n} = \frac{\vec{r}}{|r|}$$

Energy emitted per unit time and area

$$\vec{S} = \frac{c}{4\pi}(\vec{E}_\theta \times \vec{B}) = \frac{c}{4\pi}(E_\theta^2 \vec{n}) = \frac{dE}{dtdA} = \frac{dE}{dtr^2 d\Omega} = \frac{dE}{r^2\ dt sin\theta d\theta d\Phi}$$

Emitted power

$$P = -\frac{dE}{dt} = \int_0^{2\pi} d\Phi \int_0^\pi sin\theta d\theta |\vec{S}| r^2 = \frac{q^2}{4\pi c^3}|\dot{\vec{v}}|^2 \underbrace{\int_0^{2\pi} d\Phi}_{=2\pi} \underbrace{\int_0^\pi sin^3\theta d\theta}_{=\frac{4}{3}} = \boxed{\frac{2}{3}\frac{q^2}{c^3}|\dot{\vec{v}}|^2}$$

Independent of r !

Radiation from a moving charge

Larmor formula: why do we consider it for leptons only?

Emitted power depends on acceleration; subject to the same field, an electron will have a much larger acceleration

$$\left.\begin{aligned} P &= \frac{2}{3}\frac{q^2}{c^3}|\dot{\vec{v}}|^2 \\ \dot{v} &= \frac{F}{m} \end{aligned}\right\} \quad \left.\begin{aligned} P &\propto \frac{1}{m^2} \\ \frac{m_e}{m_p} &\sim \frac{1}{1836} \end{aligned}\right\} \quad \frac{P_e}{P_p} \gtrsim 10^6$$

Radiation from a moving charge

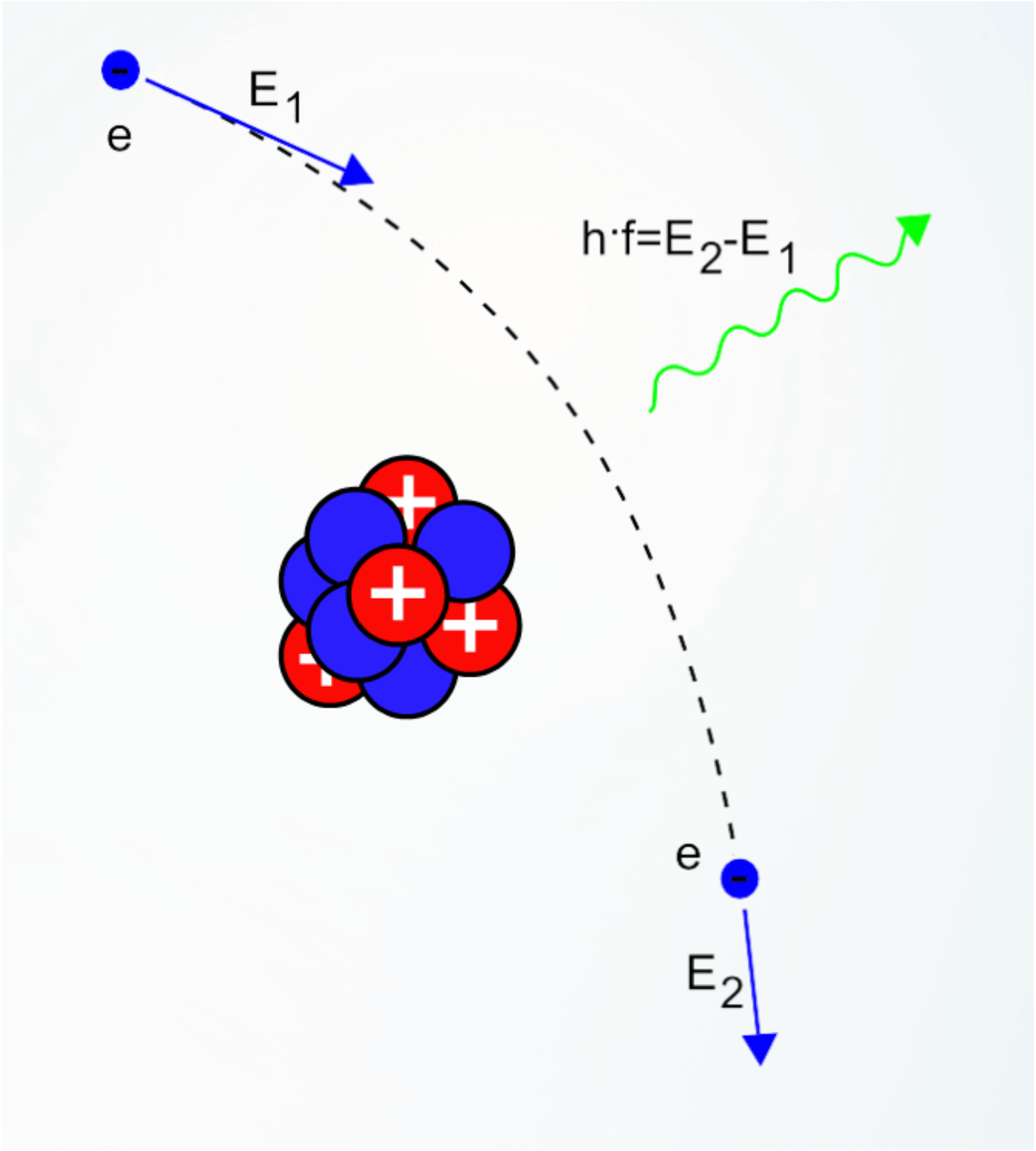
Larmor formula: when is it important?

$$P = \frac{2}{3}\frac{q^2}{c^3}\boxed{|\vec{\dot{v}}|^2}$$

Whenever we have an electron changing its velocity.

This can be due to:

- External Coulomb field (due e.g. nuclei) $\Longrightarrow$ ***bremsstrahlung*** (= braking radiation)
- External magnetic field $\Longrightarrow$ ***synchrotron*** (magnetic braking / magneto-bremsstrahlung)



Radiation from a moving charge

Larmor formula — relativistic generalization

To use the Larmor formula, we need to generalize it to the relativistic case

Consider two reference systems

K' : Moving frame with velocity $v \parallel \vec{u_x}$

Lorenz transformations

K : rest frame

Lorentz transformations among reference systems:

$$x = \Gamma(x' + vt')$$

$$ct = \Gamma(ct' + \frac{v}{c}x')$$

$$y = y'$$

$$z = z'$$

Radiation from a moving charge

Larmor formula — relativistic generalization

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Lorentz transformations among reference systems:

$$x = \Gamma(x' + vt')$$

$$ct = \Gamma\left(ct' + \frac{v}{c}x'\right)$$

$$y = y'$$

$$z = z'$$

We differentiate

$$dx = \Gamma(dx' + vdt')$$

$$dt = \Gamma dt'\left(1 + \frac{v}{c^2}\frac{dx'}{dt'}\right) = \left(1 + \frac{v}{c^2}u'_x\right)$$

Radiation from a moving charge

Larmor formula — relativistic generalization

K' : Moving frame with velocity $v \parallel \vec{u_x}$
K : rest frame

Lorenz transformations

$$x = \Gamma(x' + vt')$$

$$ct = \Gamma(ct' + \frac{v}{c}x')$$

$$dt = \Gamma dt'\left(1 + \frac{v}{c^2}\frac{dx'}{dt'}\right)$$

Velocities

$$u_x = \frac{dx}{dt} = \frac{\Gamma(dx' + vdt')}{\Gamma(dt' + \frac{v}{c^2}dx')} = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x}$$

$$u_y = \frac{dy}{dt} = \frac{dy'}{\Gamma(dt' + \frac{v}{c^2}dx')} = \frac{u'_y}{\Gamma(1 + \frac{v}{c^2}u'_x)}$$

$$u_z = \frac{dz}{dt} = \frac{dz'}{\Gamma(dt' + \frac{v}{c^2}dx')} = \frac{u'_z}{\Gamma(1 + \frac{v}{c^2}u'_x)}$$

$$u_\parallel = \frac{u'_\parallel + v}{1 + \frac{v}{c^2}u'_\parallel}$$

$$u_\perp = \frac{u'_\perp}{\Gamma(1 + \frac{v}{c^2}u'_\parallel)}$$

Radiation from a moving charge

Larmor formula — relativistic generalization

Compute the differential

$$u_\parallel = \frac{u'_\parallel + v}{1 + \frac{v}{c^2}u'_\parallel} \quad \Rightarrow \quad du_\parallel = \frac{du'_\parallel(1 + \frac{v}{c^2}u'_\parallel) - (u'_\parallel + v)(\frac{v}{c^2}du'_\parallel)}{(1 + \frac{v}{c^2}u'_\parallel)^2}$$

$$dt = \Gamma dt'\left(1 + \frac{v}{c^2}u'_\parallel\right)$$

Accelerations

$$a_\parallel = \frac{du_\parallel}{dt} = \frac{du'_\parallel(1 + \frac{v}{c^2}u'_\parallel) - (u'_\parallel + v)(\frac{v}{c^2}du'_\parallel)}{(1 + \frac{v}{c^2}u'_\parallel)^2} \frac{1}{\Gamma dt'\left(1 + \frac{v}{c^2}u'_\parallel\right)} = \frac{\overbrace{\frac{du'_\parallel}{dt'}}^{a'_\parallel}\overbrace{\left(1 - \frac{v^2}{c^2}\right)}^{\frac{1}{\Gamma^2}}}{\Gamma(1 + \frac{v}{c^2}u'_\parallel)^2} \rightarrow \boxed{a'_\parallel = \Gamma^3 a_\parallel}$$

Similarly one can derive: $\boxed{a'_\perp = \Gamma^2 a_\perp}$

Radiation from a moving charge

Larmor formula — relativistic generalization

$$P = \frac{2}{3}\frac{q^2}{c^3}|\dot{\vec{v}}|^2 \quad \Rightarrow \quad P = \frac{2}{3}\frac{q^2}{c^3}(\Gamma^2 a_\perp + \Gamma^3 a_\parallel)^2 = \frac{2}{3}\frac{q^2}{c^3}\Gamma^4(a_\perp^2 + \Gamma^2 a_\parallel^2)$$

$$a'_\parallel = \Gamma^3 a_\parallel$$

$$a'_\perp = \Gamma^2 a_\perp$$

$$a_\parallel \cdot a_\perp = 0$$

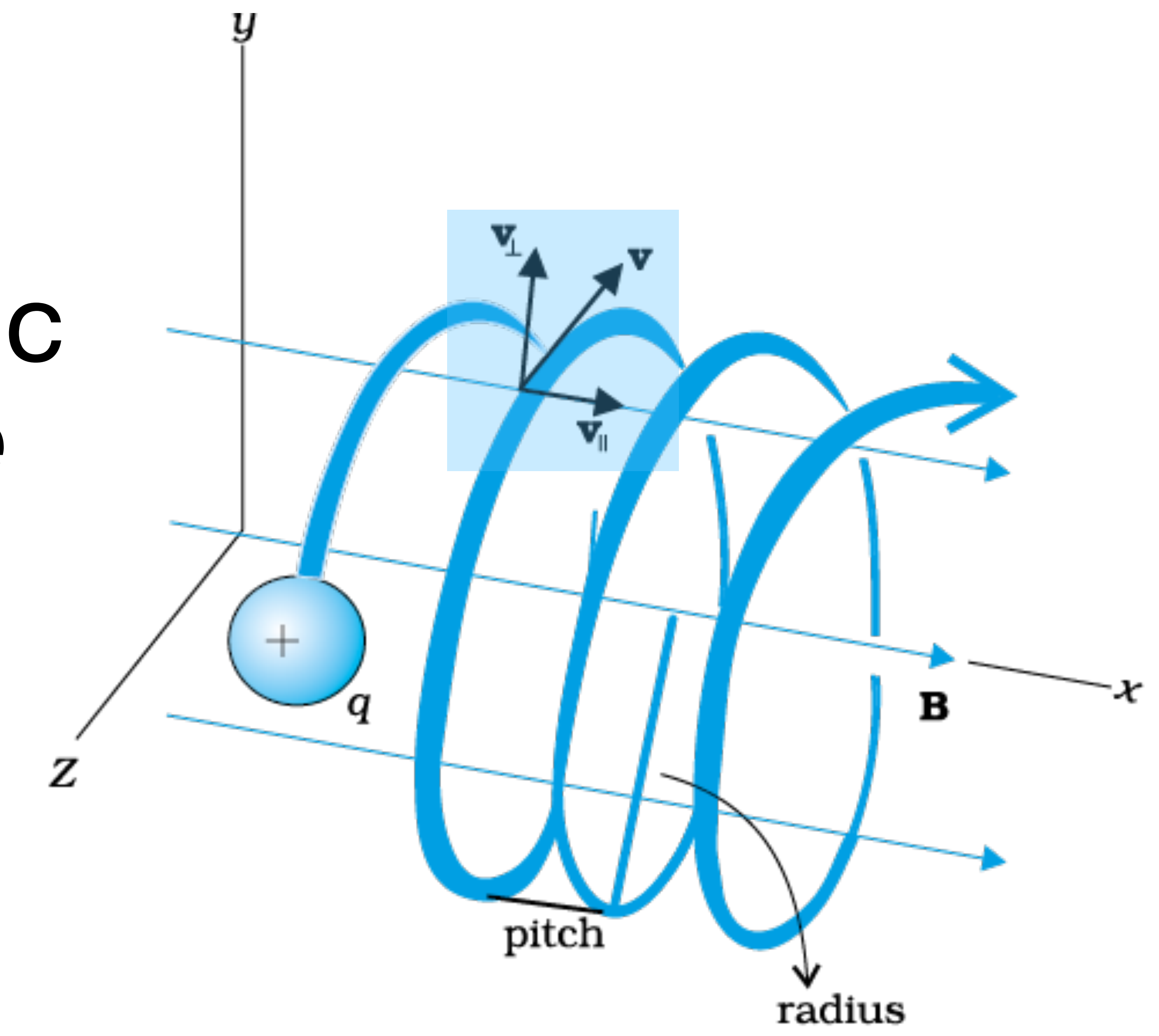
K' : Moving frame with velocity $v \parallel \overrightarrow{u_x}$
K : lab frame

Synchrotron emission

From the Larmor formula to radiation

$$P = \frac{2}{3}\frac{q^2}{c^3}\Gamma^4(a_\perp^2 + \Gamma^2 a_\parallel^2)$$

Synchrotron: electric charges gyrating due to a magnetic field change their trajectory and therefore radiate —> *we have to compute the acceleration linked to this motion*

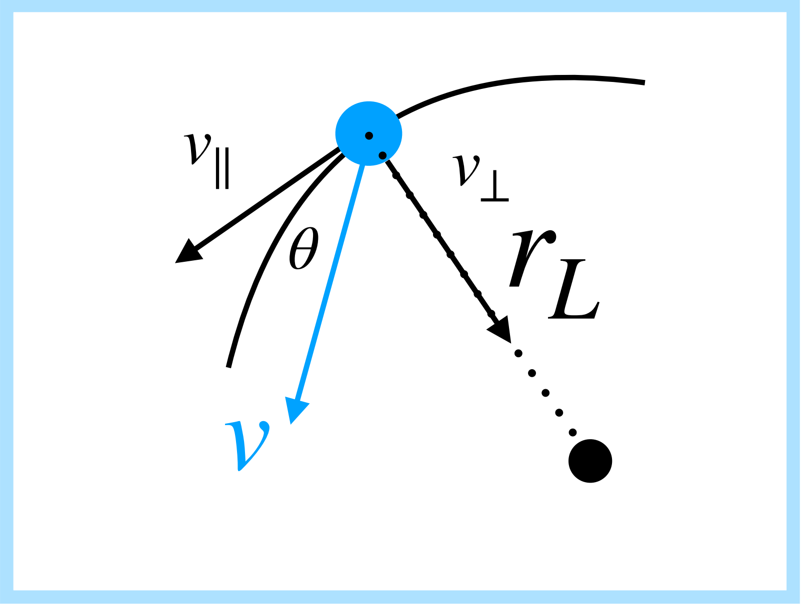


$v_\parallel$ is the component parallel to the magnetic field

θ is the angle between B and v

Particles in a B field gyrate with Larmor frequency $\Omega_L = \dfrac{qB}{\Gamma mc}$

Which is associated to the Larmor radius $r_L = \dfrac{v_\perp}{\Omega_L}$



Synchrotron emission

From the Larmor formula to radiation

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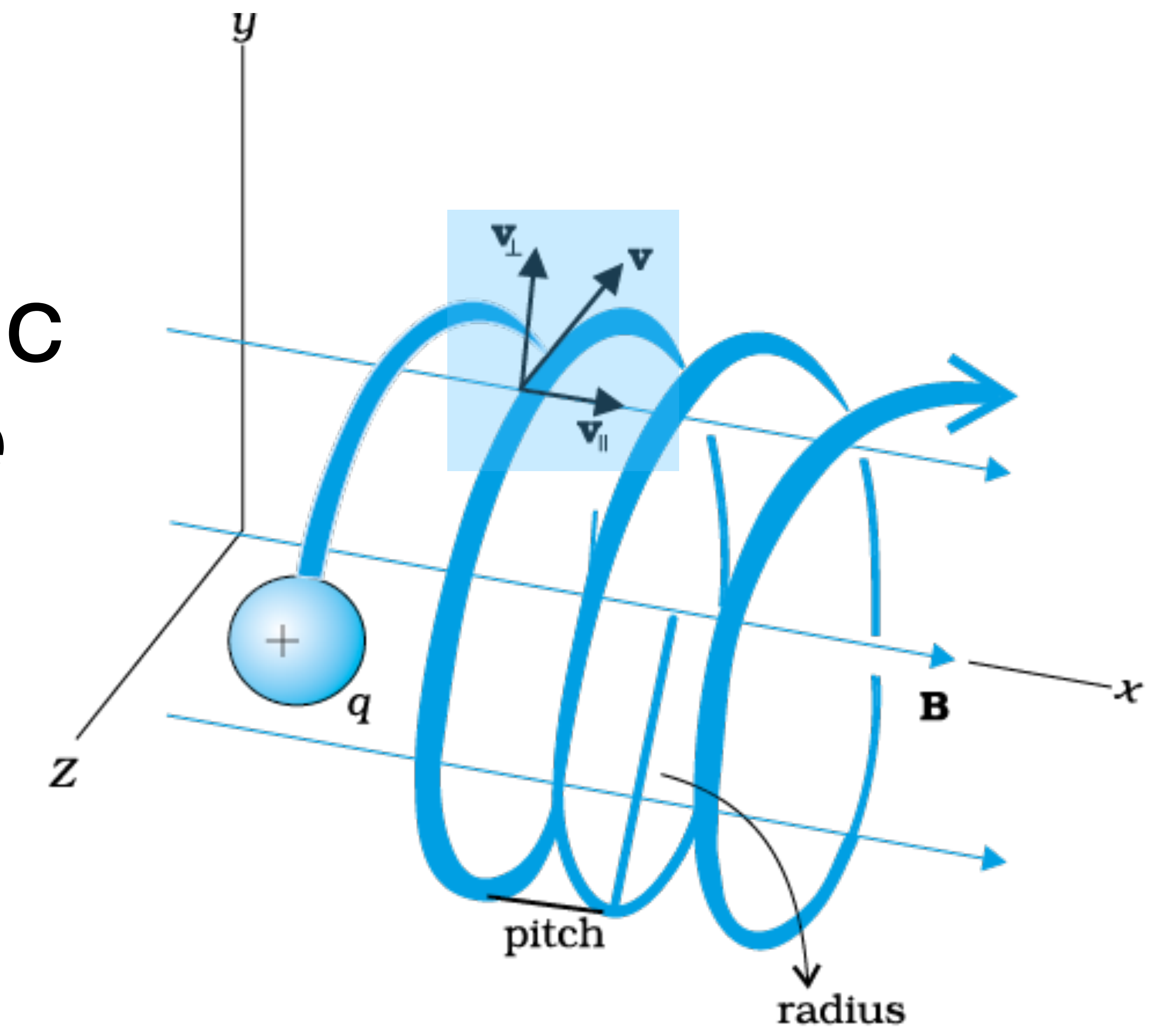

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θ is the angle between B and v

Particles in a B field gyrate with Larmor frequency $\Omega_L = \dfrac{qB}{\Gamma mc}$

Which is associated to the Larmor radius $r_L = \dfrac{v_\perp}{\Omega_L}$

Acceleration turns out:

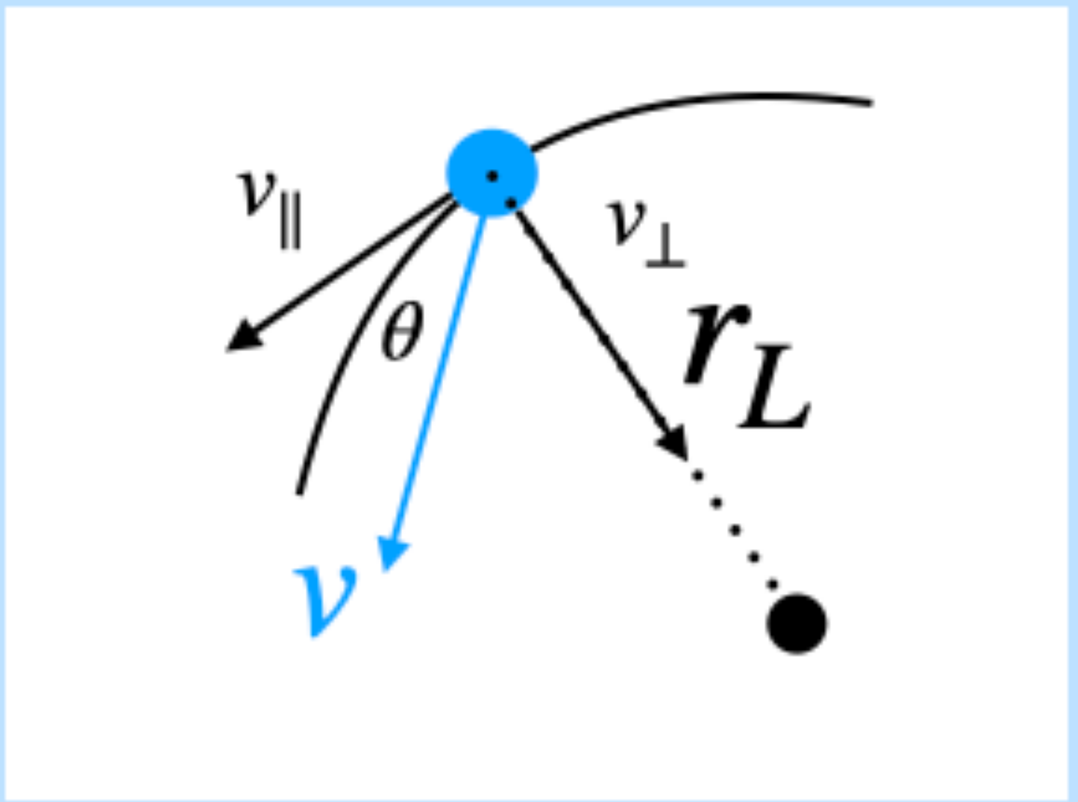
$$a_\perp = \frac{v_\perp^2}{r_L} = \Omega_L v_\perp$$

$$a_\parallel = 0$$

Synchrotron emission

From the Larmor formula to radiation

$$P = \frac{2}{3}\frac{q^2}{c^3}\Gamma^4(a_\perp^2 + \Gamma^2 \cancel{a_\parallel^2}) \qquad \text{(A)}$$



Synchrotron: electric charges gyrating due to a magnetic field change their trajectory and therefore radiate as a consequence of acceleration in the perpendicular direction

$$\Omega_L = \frac{qB}{\Gamma mc}$$

$$r_L = \frac{v_\perp}{\Omega_L} = \frac{m\Gamma vc}{qB}$$

$$a_\perp = \frac{v_\perp^2}{r_L} = \Omega_L v_\perp$$

$$a_\parallel = 0$$

$$\text{(A)} \Rightarrow P = \frac{2}{3}\frac{e^2\Gamma^4 v_\perp^2 e^2 B^2}{c^3\Gamma^2 m^2 c^2} = \frac{2}{3}\frac{\beta^2 e^4 B^2}{m^2 c^3}\Gamma^2 \sin^2\theta$$

$$v_\perp = v\sin\theta$$

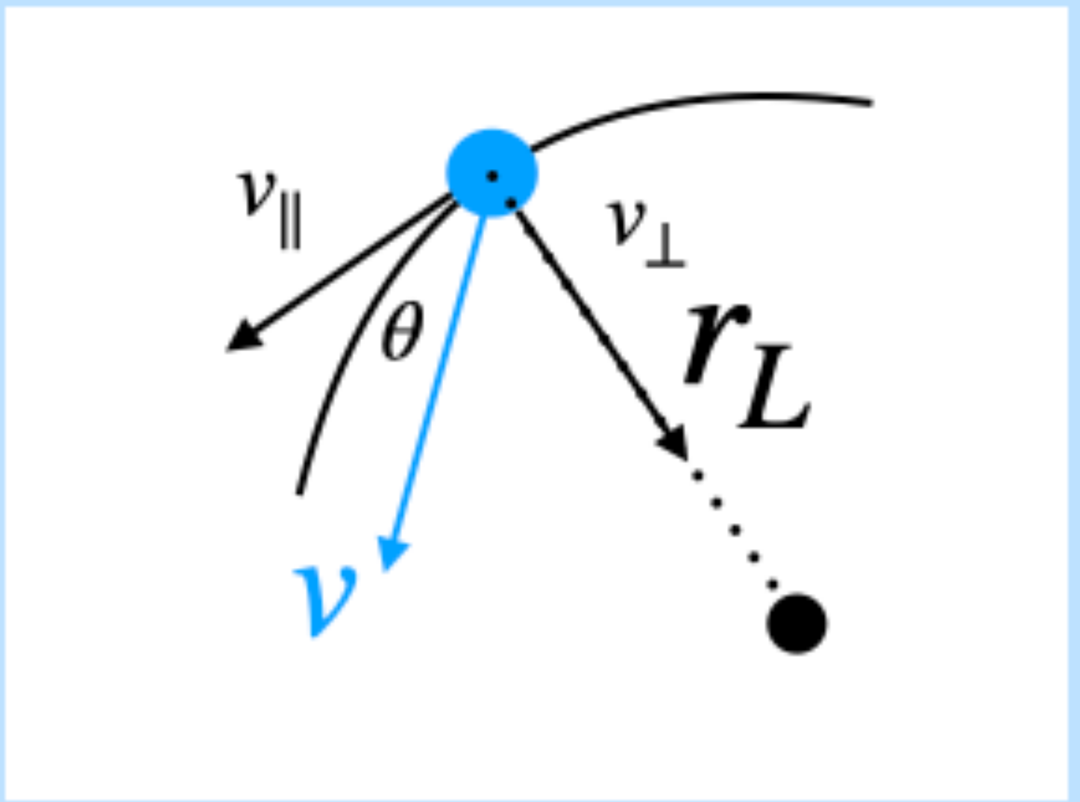
$$\frac{v}{c} = \beta$$

Synchrotron emission

From the Larmor formula to radiation

$$P = \frac{2}{3}\frac{q^2}{c^3}\Gamma^4(a_\perp^2 + \Gamma^2 \cancel{a_\parallel^2}) \quad \text{(A)}$$

Synchrotron: electric charges gyrating due to a magnetic field change their trajectory and therefore radiate as a consequence of acceleration in the perpendicular direction



$$\Omega_L = \frac{qB}{\Gamma mc}$$

$$r_L = \frac{v_\perp}{\Omega_L} = \frac{m\Gamma vc}{qB}$$

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$$a_\parallel = 0$$

(A) ➡ $$P = \frac{2}{3}\frac{e^2\Gamma^4 v_\perp^2 e^2 B^2}{c^3\Gamma^2 m^2 c^2} = \frac{2}{3}\frac{\beta^2 e^4 B^2}{m^2 c^3}\Gamma^2 \boxed{\sin^2\theta}$$

Depends on the relative direction of velocity and B

$$v_\perp = v\sin\theta$$

$$\frac{v}{c} = \beta$$

Synchrotron emission

From the Larmor formula to radiation

$$P_{sync} = \frac{2}{3}\frac{\beta^2 e^4 B^2}{m^2 c^3}\Gamma^2 \sin^2\theta$$

$$\langle \sin^2\theta \rangle = \frac{1}{4\pi}\int_0^{2\pi} d\phi d\theta \int_0^{\pi} \sin^2\theta sin\theta = \frac{2}{3}$$

Average over the angles

Synchrotron emission

From the Larmor formula to radiation

$$P_{sync} = \frac{2}{3}\frac{\beta^2 e^4 B^2}{m^2 c^3}\Gamma^2 \sin^2\theta$$

$$\langle \sin^2\theta \rangle = \frac{1}{4\pi}\int_0^{2\pi} d\phi d\theta \int_0^{\pi} \sin^2\theta sin\theta = \frac{2}{3}$$

$$\langle P_{sync} \rangle = \frac{4}{9}\frac{\beta^2 e^4 B^2}{m^2 c^3}\Gamma^2 = \frac{4}{3}c\frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2\left(\frac{m_e}{m}\right)^2\frac{B^2}{8\pi}$$

Synchrotron emission

From the Larmor formula to radiation

$$P_{sync} = \frac{2}{3}\frac{\beta^2 e^4 B^2}{m^2 c^3}\Gamma^2 \sin^2\theta$$

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Proton synchrotron is negligible compared to electron synchrotron

Synchrotron emission

From the Larmor formula to radiation

$$P_{sync} = \frac{2}{3}\frac{\beta^2 e^4 B^2}{m^2 c^3}\Gamma^2 \sin^2\theta$$

$$\langle \sin^2\theta \rangle = \frac{1}{4\pi}\int_0^{2\pi} d\phi d\theta \int_0^{\pi} \sin^2\theta sin\theta = \frac{2}{3}$$

$$\langle P_{sync} \rangle = \frac{4}{9}\frac{\beta^2 e^4 B^2}{m^2 c^3}\Gamma^2 = \frac{4}{3}c\underbrace{\frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2}_{\text{Thomson cross-section}}\left(\frac{m_e}{m}\right)^2\underbrace{\frac{B^2}{8\pi}}_{\text{Magnetic energy density}}\Gamma^2 = \frac{4}{3}c\sigma_T\left(\frac{m_e}{m}\right)^2\Gamma^2 u_B$$

$$\boxed{\langle P_{sync} \rangle \propto \Gamma^2 u_B \propto \Gamma^2 B^2}$$

Synchrotron emission

Synchrotron cooling

$$\langle P_{sync} \rangle = \frac{4}{3} c \sigma_T \left(\frac{m_e}{m} \right)^2 \Gamma^2 u_B$$

$$t_{cool}(\Gamma) = \frac{E}{|dE/dt|}$$

Synchrotron emission

Synchrotron cooling

$$\langle P_{sync} \rangle = \frac{4}{3} c \sigma_T \left(\frac{m_e}{m} \right)^2 \Gamma^2 u_B$$

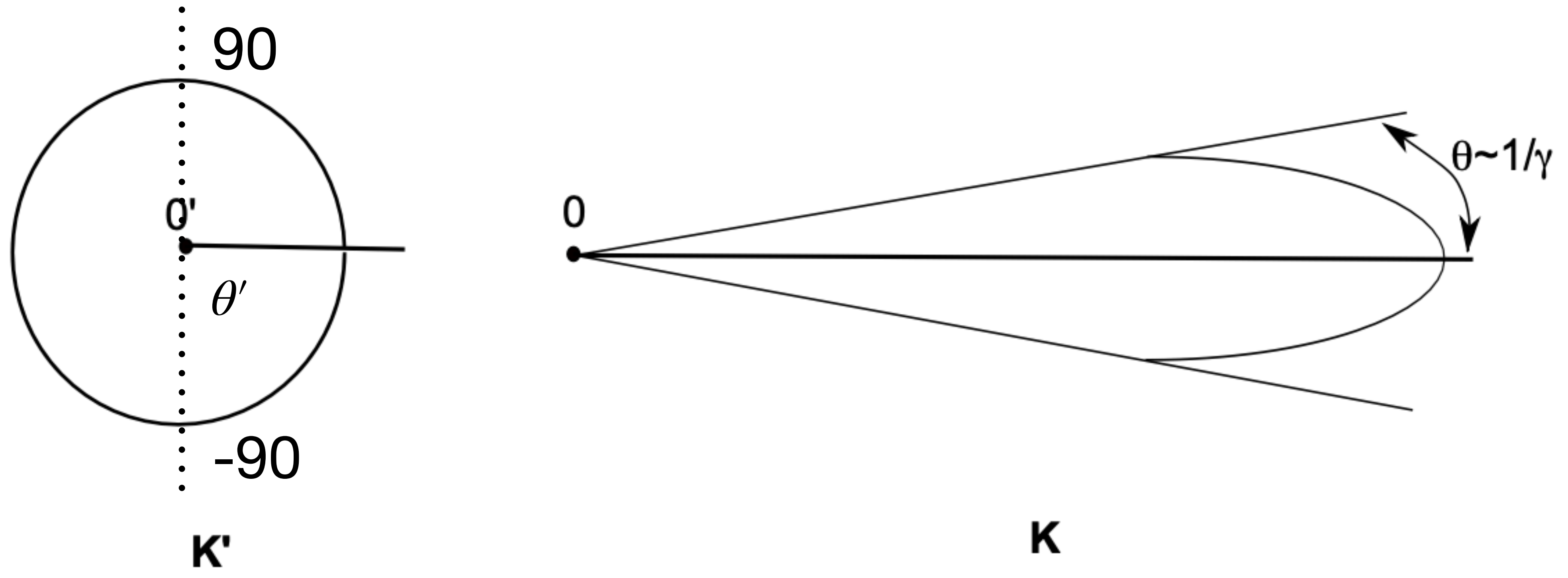
$$t_{cool}(\Gamma) = \frac{E}{|dE/dt|} = \frac{\Gamma m_e c^2}{\langle P_{sync} \rangle} \propto \boxed{\Gamma^{-1}} = 7.8 \times 10^8 \frac{1}{B^2 \Gamma} \text{ s}$$

Synchrotron emission

Relativistic Beaming

K' : Moving frame with velocity $v \parallel \vec{u_{\parallel}}$ (emitter)

K : rest frame (observer)



$$\tan\theta = \frac{u_\perp}{u_\parallel} = \frac{u'_\perp}{\Gamma(u'_\parallel + v)} = \frac{u'\sin\theta'}{\Gamma(u'\cos\theta' + v)}$$

For light: $u' = c$

$$= \frac{\sin\theta'}{\Gamma(\cos\theta' + \frac{v}{c})}$$

$|\theta'| < 90^\circ$

For relativistic particles

$v \sim c$

$$|\tan\theta| < \frac{1}{\Gamma(\frac{v}{c})} \rightarrow |\theta| \lesssim \frac{1}{\Gamma}$$

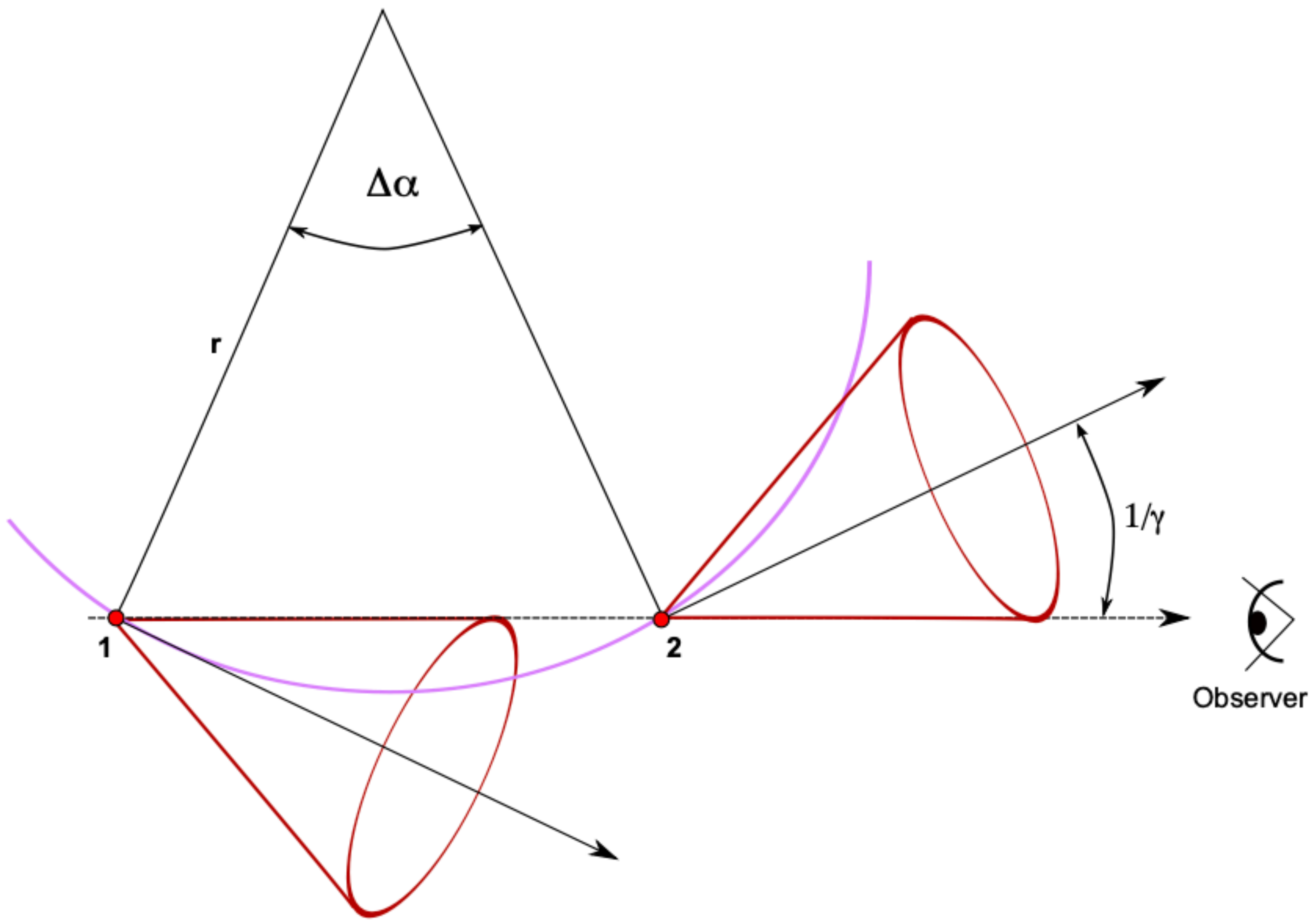
In the observer frame the emission is beamed due to relativistic effects: the cone is large $2/\Gamma$

$$u_\parallel = \frac{u'_\parallel + v}{1 + \frac{v}{c^2}u'_\parallel} \quad u_\perp = \frac{u'_\perp}{\Gamma(1 + \frac{v}{c^2}u'_\parallel)}$$

Synchrotron emission

Characteristic frequency

Emission is beamed due to relativistic effects



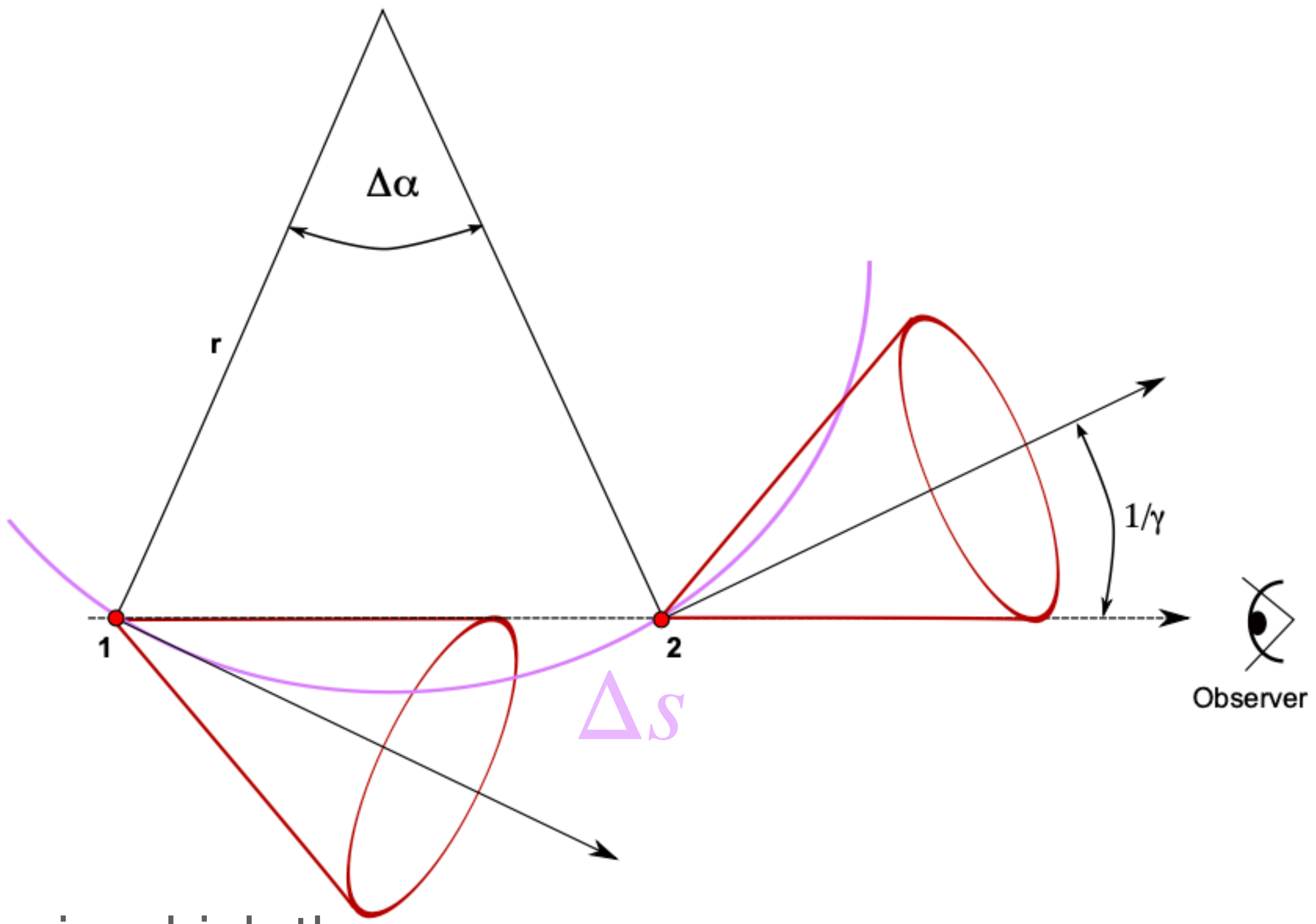
Observer only sees radiation once per obit when particle velocity is within $2/\Gamma$ of line of sight.
Δt_{obs}:=duration for which beam stays within line of sight

- Observer sees narrow pulse of duration Δt_{obs}
- Most of the radiation power **must appear at a characteristic frequency** $\omega_{ch} = 2\pi\nu_{ch} = \dfrac{1}{\Delta t_{obs}}$

Synchrotron emission

Characteristic frequency

Emission is beamed due to relativistic effects



Space in which the radiation is visible

Observer only sees radiation once per obit when particle velocity is within $2/\Gamma$ of line of sight.
Δt_{obs}:=duration for which beam stays within line of sight

- Observer sees narrow pulse of duration Δt_{obs}
- Most of the radiation power **must appear at a characteristic frequency** $\omega_{ch} = 2\pi\nu_{ch} = \dfrac{1}{\Delta t_{obs}}$

$$\Delta s = r_L \Delta\alpha = r_L \frac{2}{\Gamma}$$

$$\Gamma m \frac{v^2}{r_L} = \frac{q}{c} B v \sin\theta \rightarrow v = \frac{qB\Gamma\Delta s}{2\Gamma mc}$$

$$\Delta t_{obs} = \frac{\Delta s}{v} - \frac{\Delta s}{c} = \frac{2mc}{qB} - \frac{2mv}{qB} = \frac{2mc}{qB}\left(1 - \frac{v}{c}\right)\frac{1+\frac{v}{c}}{1+\frac{v}{c}} \simeq \frac{2mc}{\Gamma^2 qB}$$

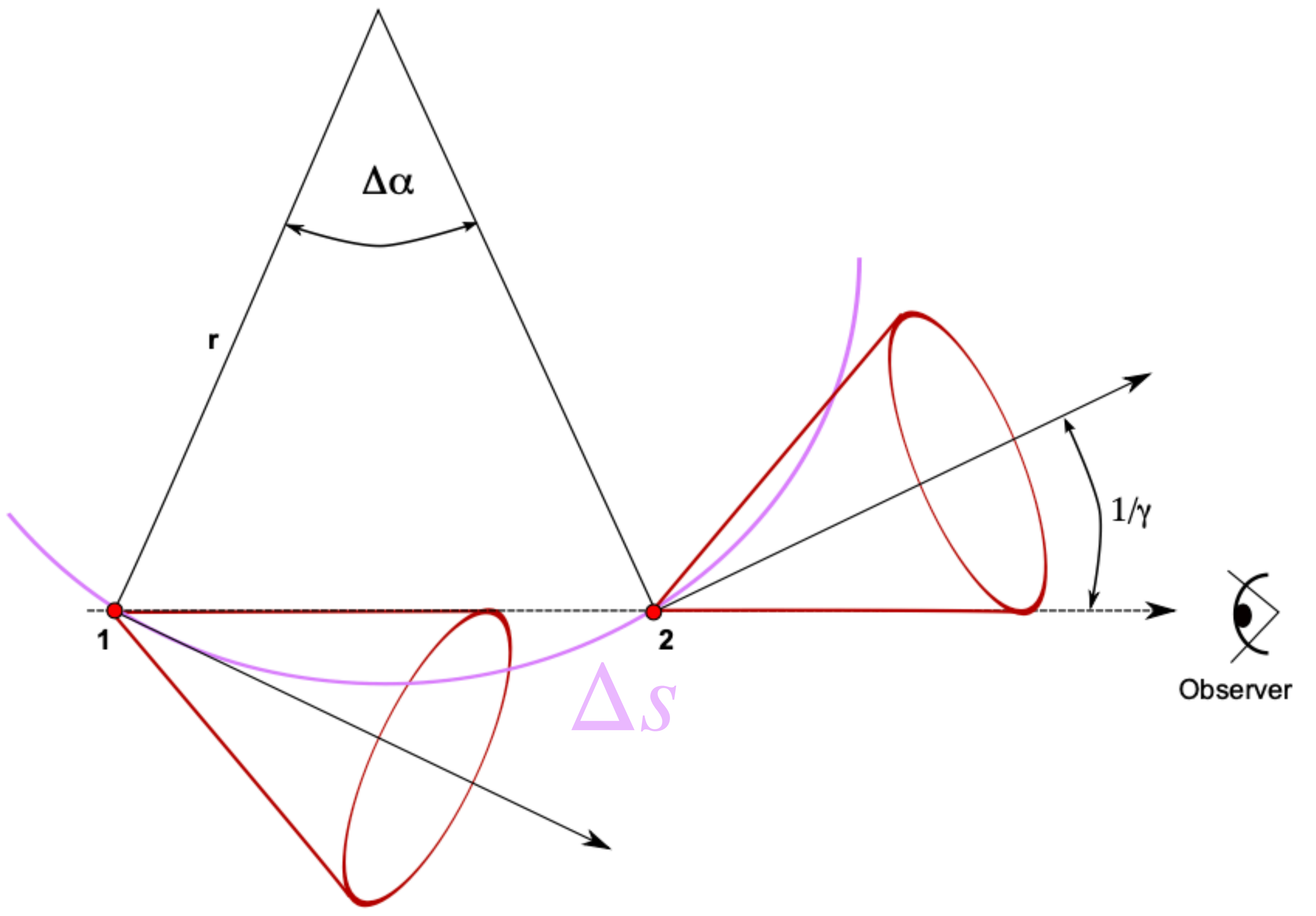
Time during which the radiation is visible

Signal propagation time

Synchrotron emission

Characteristic frequency

Emission is beamed due to relativistic effects



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- Observer sees narrow pulse of duration Δt_{obs}
- Most of the radiation power **must appear at a characteristic frequency** $\omega_{ch} = 2\pi\nu_{ch} = \dfrac{1}{\Delta t_{obs}}$

$$\Delta t_{obs} \simeq \frac{2mc}{\Gamma^2 qB} \rightarrow \boxed{\omega_{ch} \simeq \frac{1}{\Delta t_{obs}} = \Gamma^2 \frac{qB\sin\theta}{mc}}$$

This is the characteristic frequency at which an electron of energy Γ *emits*

Synchrotron emission

From the Larmor formula to radiation

We have the emitted power in unit time but we need to turn it into something we can measure: emitted power per unit frequency, namely **the spectrum,** P_ν

We need to find a function such that:

$$\int_0^\infty P_\nu(\Gamma)d\nu = P_{sync} \quad \text{[Larmor]}$$

$$P_\nu(\Gamma) = P_{sync}\tilde{F}\left(\frac{\nu}{\nu_c}\right)$$

We can factorize the dependence on ν

$$\frac{1}{\nu_c}\int_0^\infty \tilde{F}\left(\frac{\nu}{\nu_c}\right) = 1$$

Synchrotron emission

From the Larmor formula to radiation

We have the emitted power in unit time but we need to turn it into something we can measure: emitted power per unit frequency, namely **the spectrum,** $P(\nu)$

$$\int_0^{\infty} P_\nu(\Gamma) d\nu = P_{sync}$$

$$P_\nu(\Gamma) = P_{sync} \tilde{F}\left(\frac{\nu}{\nu_c}\right)$$

$$\frac{1}{\nu_c}\int_0^{\infty} \tilde{F}\left(\frac{\nu}{\nu_c}\right) = 1$$

Electric field $\epsilon(t) \propto g(\omega_c t)$

$$\hat{\epsilon}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \epsilon(t) e^{i\omega t} dt$$

$$\epsilon(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{\epsilon}(\omega) e^{-i\omega t} d\omega$$

Now we consider the equation of the radiated electric field $\epsilon(t) \propto g(\omega_c t)$ (from the full-non approximated Larmor formula, derived from Lienard-Wiechert potentials and its inverse Fourier transformation

Synchrotron emission

From the Larmor formula to radiation

We have the emitted power in unit time but we need to turn it into something we can measure: emitted power per unit frequency, namely **the spectrum,** $P(\nu)$

$$\int_0^{\infty} P_\nu(\Gamma) d\nu = P_{sync}$$

$$P_\nu(\Gamma) = P_{sync} \tilde{F}\left(\frac{\nu}{\nu_c}\right)$$

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$$\epsilon(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{\epsilon}(\omega) e^{-i\omega t} d\omega$$

Parseval theorem: $\int_{-\infty}^{+\infty} \epsilon^2(t) dt = 2\pi \int_{-\infty}^{+\infty} |\hat{\epsilon}(\omega)|^2 d\omega$

Poynting theorem: $\nabla \cdot \mathbf{S} = -\frac{\partial u_E}{\partial t}$

Poynting vector

Energy density

We consider now how the energy density is linked to the Poynting vector

$$\frac{c\epsilon^2}{4\pi\Delta x} \simeq \frac{E}{\Delta V \Delta t} \rightarrow \frac{dE}{dA} = \frac{c}{4\pi}\int dt \epsilon^2(t)$$

$$\frac{dE}{dA} = \frac{c}{4\pi}\int dt \epsilon^2(t) = c\int_0^{\infty} |\hat{\epsilon}(\omega)|^2 d\omega$$

$$\frac{dE}{dA dt d\omega} = \frac{c}{T}\frac{dE}{dA d\omega} = \frac{c}{T}|\hat{\epsilon}(\omega)|^2$$

Synchrotron emission

From the Larmor formula to radiation

We have the emitted power in unit time but we need to turn it into something we can measure: emitted power per unit frequency, namely **the spectrum,** $P(\nu)$

$$\int_0^{\infty} P_\nu(\Gamma) d\nu = P_{sync}$$

$$P_\nu(\Gamma) = P_{sync}\tilde{F}\left(\frac{\nu}{\nu_c}\right)$$

$$\frac{1}{\nu_c}\int_0^{\infty}\tilde{F}\left(\frac{\nu}{\nu_c}\right) = 1$$

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Poynting vector

Poynting theorem: $\nabla\cdot\mathbf{S} = -\frac{\partial u_E}{\partial t}$ Energy density

$$\left.\begin{array}{l} \frac{c\epsilon^2}{4\pi\Delta x} \simeq \frac{E}{\Delta V\Delta t} \rightarrow \frac{dE}{dA} = \frac{c}{4\pi}\int dt\epsilon^2(t) \\ \frac{dE}{dA} = \frac{c}{4\pi}\int dt\epsilon^2(t) = c\int_0^{\infty}|\hat{\epsilon}(\omega)|^2 d\omega \end{array}\right\}$$

$$\frac{dE}{dAdtd\omega} = \frac{c}{T}\frac{dE}{dAd\omega} = \frac{c}{T}|\hat{\epsilon}(\omega)|^2$$

Need modified-Bessel function…

$$P_\nu = 2\pi P_\omega(\Gamma) = 2\pi\frac{c}{T}\int\frac{dE}{dAdtd\omega}r^2 d\Omega = 2\pi\frac{c}{T}\int r^2|\hat{\epsilon}(\omega)|^2 d\Omega = \cancel{2\pi}\frac{c\Omega_L}{\cancel{2\pi}}\int r^2|\hat{\epsilon}(\omega)|^2 d\Omega = \frac{qB}{\Gamma m}\int r^2|\hat{\epsilon}(\omega)|^2 d\Omega$$

Synchrotron emission

Single particle spectrum

Ghisellini 2012

One electron emits this power:

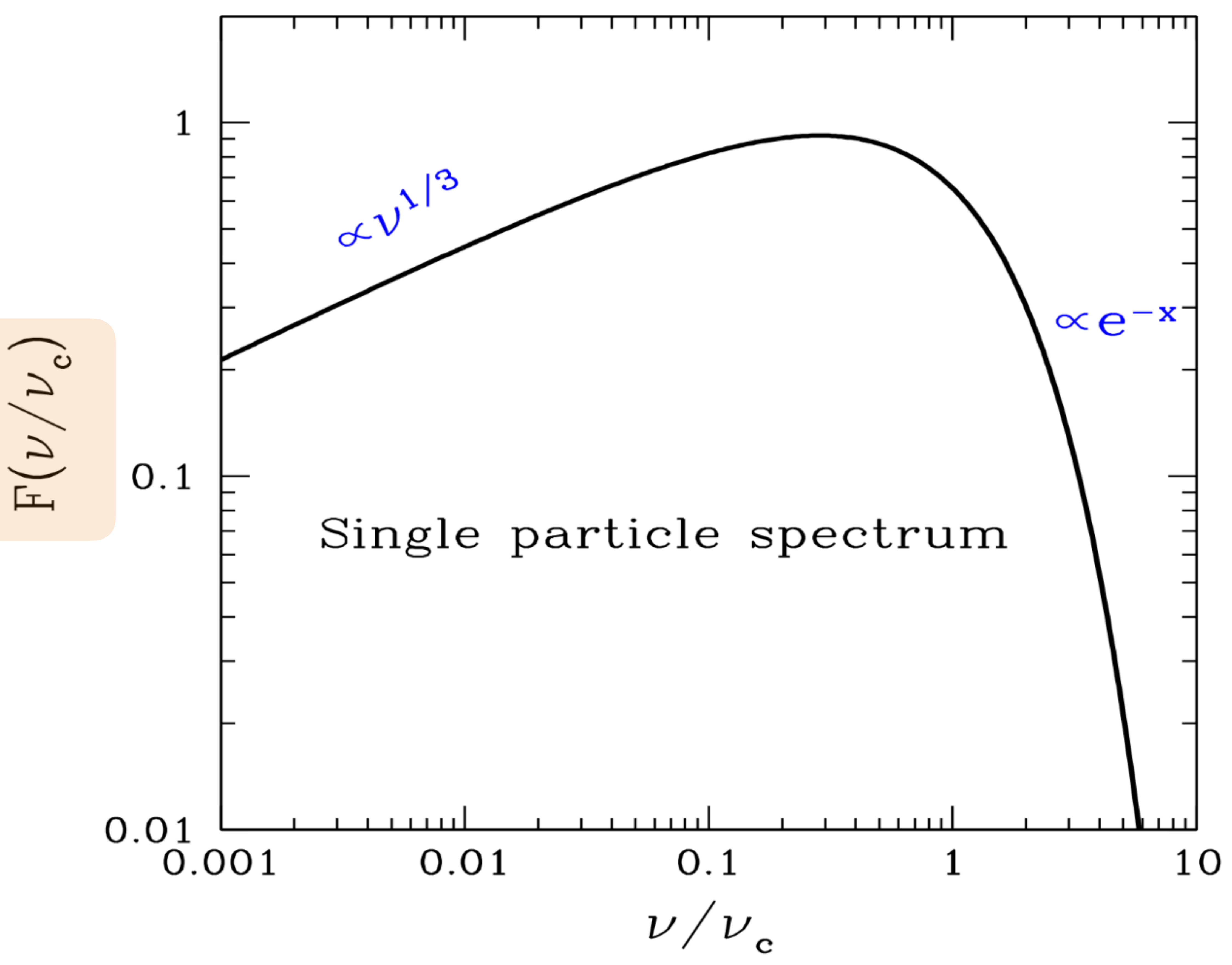
$$P_\nu(\Gamma) = \frac{qB}{\Gamma m}\int r^2 |\hat{\epsilon}(\omega)|^2 d\Omega = \frac{\sqrt{3}e^3 B \sin\theta}{mc^2} F\left(\frac{\nu}{\nu_c}\right)$$

..modified-Bessel function:

$$F\left(\frac{\nu}{\nu_c}\right) \equiv \frac{\nu}{\nu_c}\int_{\nu/\nu_c}^{\infty} K_{5/3}(y)dy \simeq 1.8\left(\frac{\nu}{\nu_c}\right)^{0.3} e^{-\frac{\nu}{\nu_c}}$$

$$\nu_c = \frac{3}{4\pi}\Gamma^2\frac{qB\sin\theta}{mc} = \frac{3}{2}\nu_{ch}$$

$$\nu_{ch} = 2\pi\omega_{ch} = 2\pi\Gamma^2\frac{qB\sin\theta}{mc}$$



Synchrotron emission

Single particle spectrum

$$P_\nu(\Gamma) = \frac{qB}{\Gamma m}\int r^2 |\hat{\epsilon}(\omega)|^2 d\Omega = \frac{\sqrt{3}e^3 B\sin\theta}{mc^2}F\left(\frac{\nu}{\nu_c}\right)$$

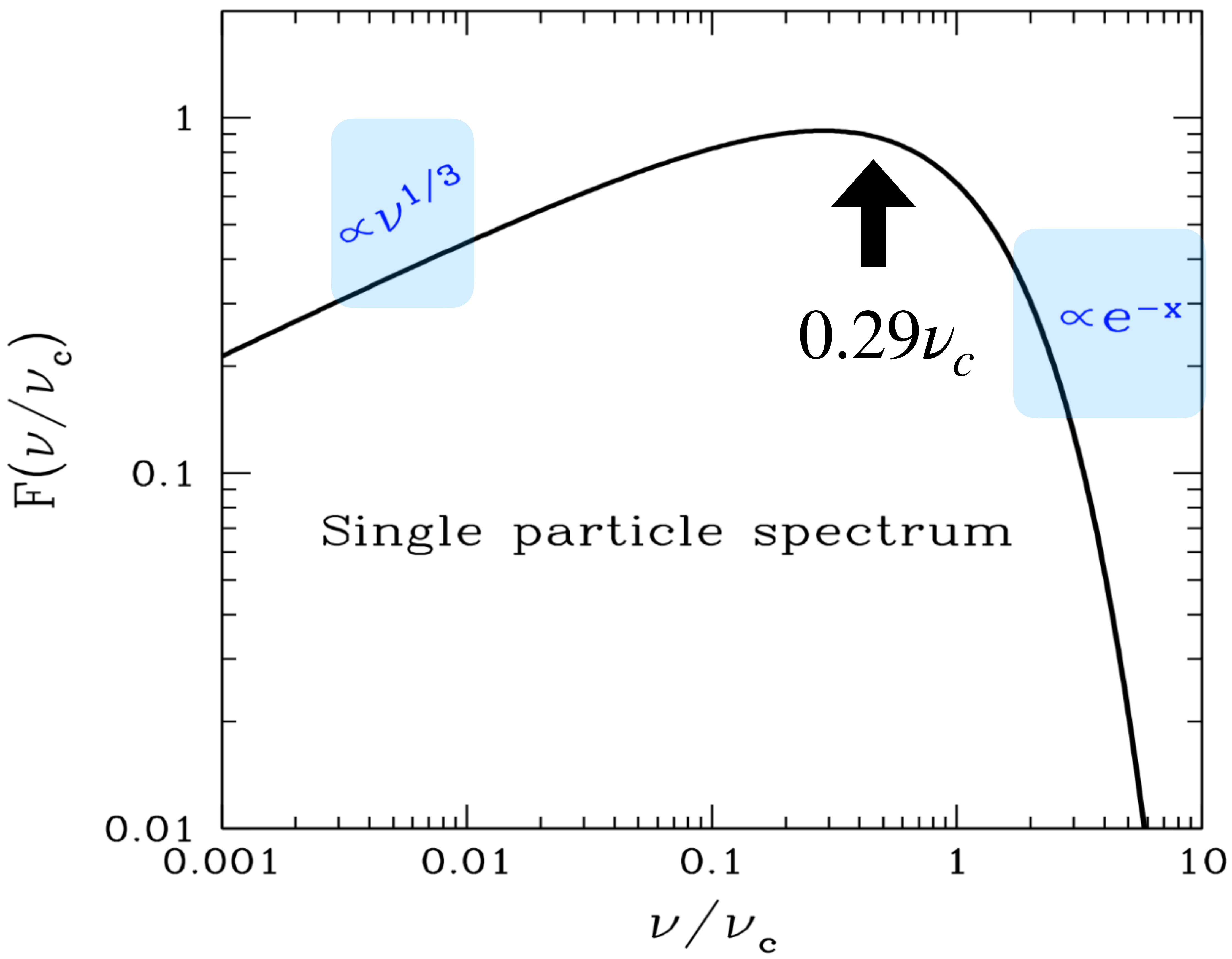
$$F\left(\frac{\nu}{\nu_c}\right) \equiv \frac{\nu}{\nu_c}\int_{\nu/\nu_c}^{\infty} K_{5/3}(y)dy \simeq 1.8\left(\frac{\nu}{\nu_c}\right)^{0.3} e^{-\frac{\nu}{\nu_c}}$$

$$\nu_c = \frac{3}{4\pi}\Gamma^2\frac{qB\sin\theta}{mc} = \frac{3}{2}\nu_{ch} \equiv \boxed{\Gamma^2\nu_L}$$

$$\nu_{ch} = 2\pi\omega_{ch} = 2\pi\Gamma^2\frac{qB\sin\theta}{mc}$$

$$\omega_L = \frac{qBsin\theta}{\Gamma mc}$$

Ghisellini 2012



The spectrum has always this shape, and scales and shift with ν_c depending on Γ

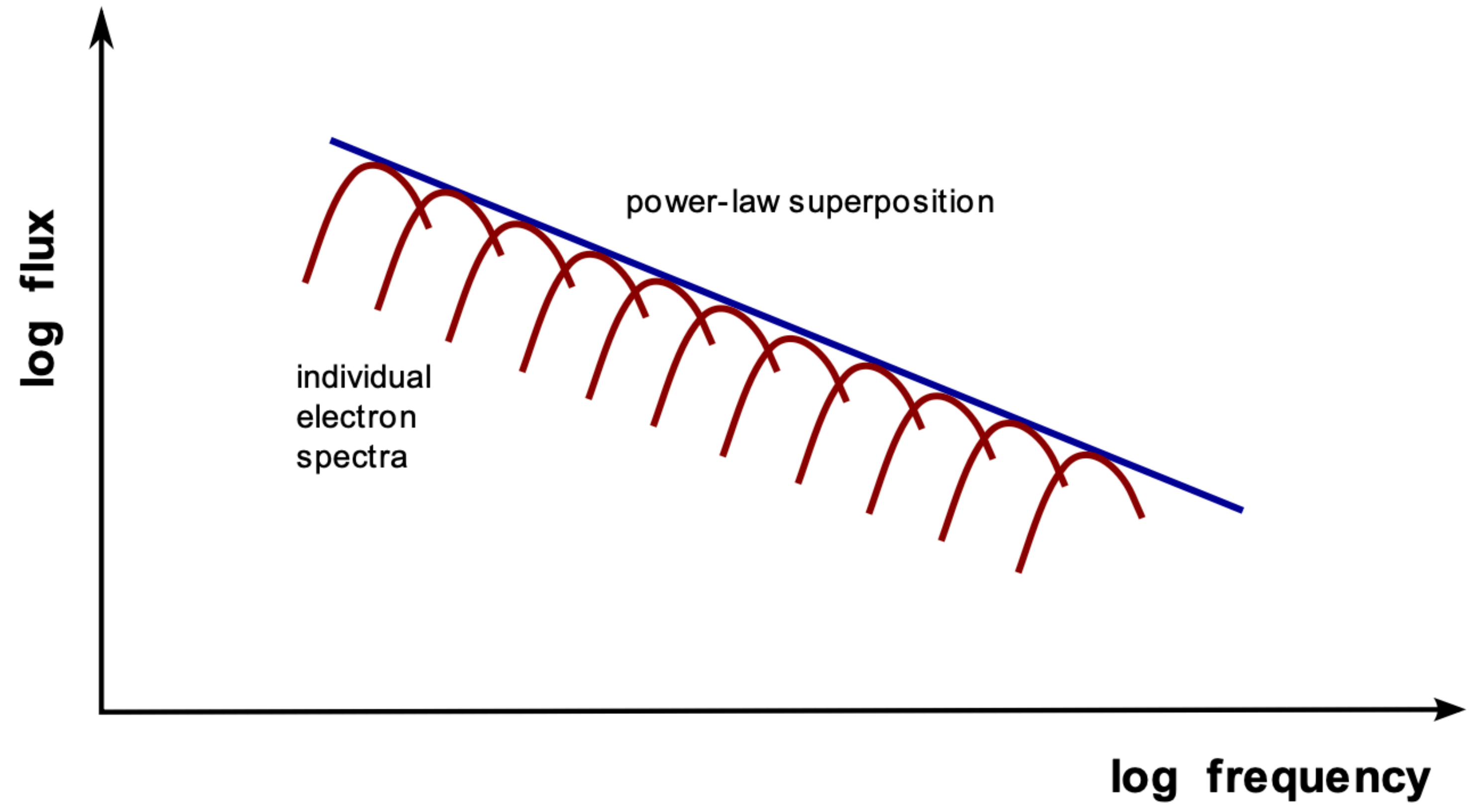
Synchrotron emission

Many electrons

Power law distribution $$N_e(\Gamma)d\Gamma = N_{e,0}\Gamma^{-p}d\Gamma$$

We have to Integrate over the emitted spectrum of each electron

The single particle spectrum has always the same shape, and scales and shift with ν_c



Synchrotron emission

Many electrons

$$\langle P_{sync}\rangle = \frac{4}{3}c\sigma_T\left(\frac{m_e}{m}\right)^2\Gamma^2 u_B$$

Power law distribution $N_e(\Gamma)d\Gamma = N_{e,0}\Gamma^{-p}d\Gamma$

δ- approximation

$$\nu_{ch} = \Gamma^2\nu_L$$

Integrate over the emitted spectrum of each electron: $j_\nu = \int_{\Gamma_{min}}^{\Gamma_{max}} \langle P_\nu(\Gamma)\rangle N_e(\Gamma)d\Gamma \approx \int_{\Gamma_{min}}^{\Gamma_{max}} N_e(\Gamma)\langle P_{syn}\rangle\delta(\nu - \Gamma^2\nu_L)d\Gamma$

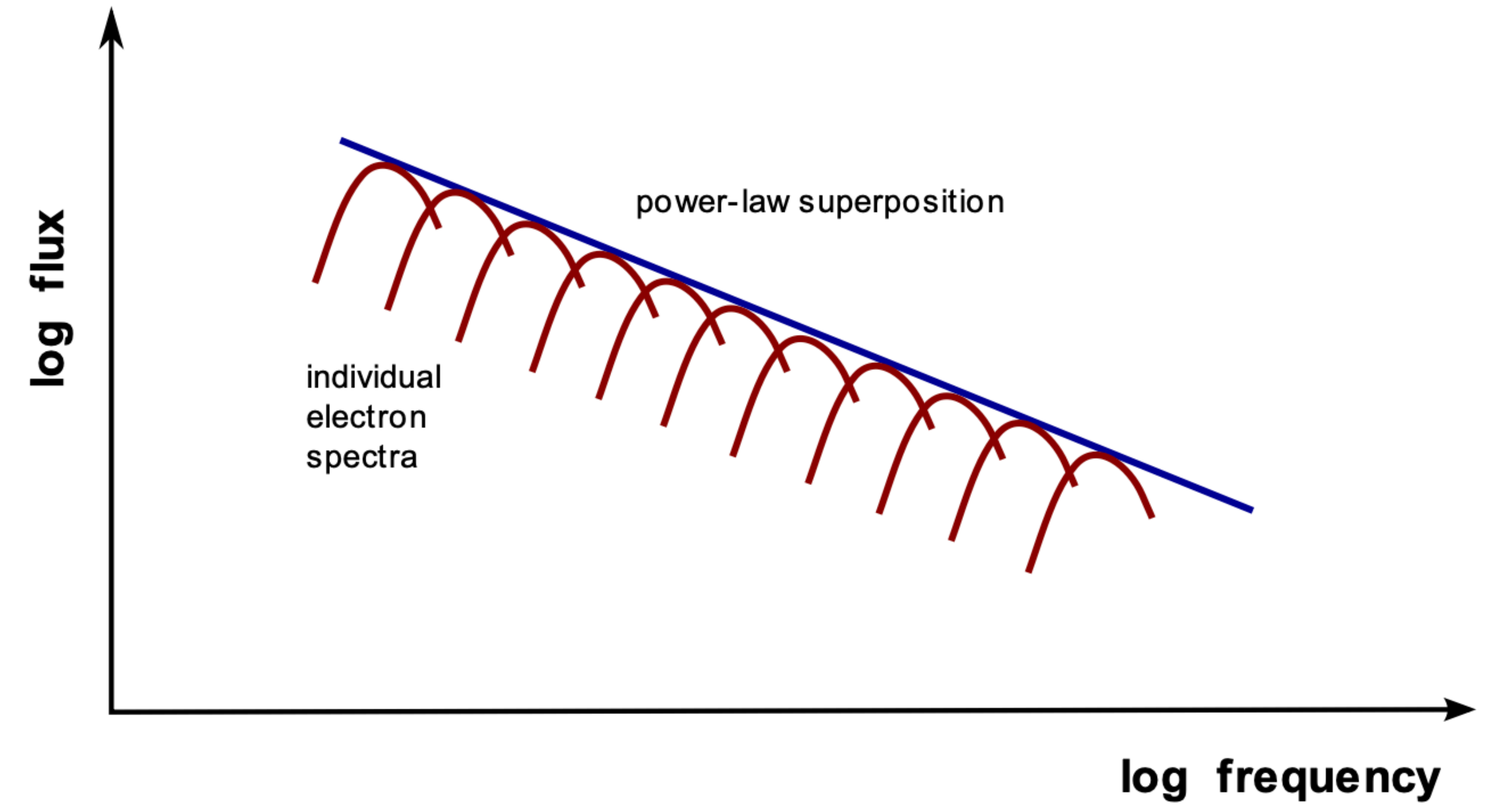
$$\simeq \frac{4}{3}c\sigma_T u_B N_{e,0}\int_{\Gamma_{min}}^{\Gamma_{max}}\frac{\Gamma^2}{\Gamma^p}\delta(\nu - \Gamma^2\nu_L)$$

The single particle spectrum has always the same shape, and scales and shift with ν_c

Now we change from Γ to ν' using $\nu' = \Gamma^2\nu_L$ hence $d\Gamma = \frac{d\nu'}{(2\Gamma\nu_L)}$

$$= \frac{4}{6\nu_L}c\sigma_T u_B N_{e,0}\int_{\Gamma^2_{min}\nu_L}^{\Gamma^2_{max}\nu_L}\left(\frac{\nu_L}{\nu'}\right)^{\frac{(p-1)}{2}}\delta(\nu - \nu')d\nu'$$

$$= \frac{2}{3\nu_L}c\sigma_T u_B N_{e,0}\left(\frac{\nu_L}{\nu'}\right)^{\frac{(p-1)}{2}} \propto \nu^{\frac{-(p-1)}{2}}$$

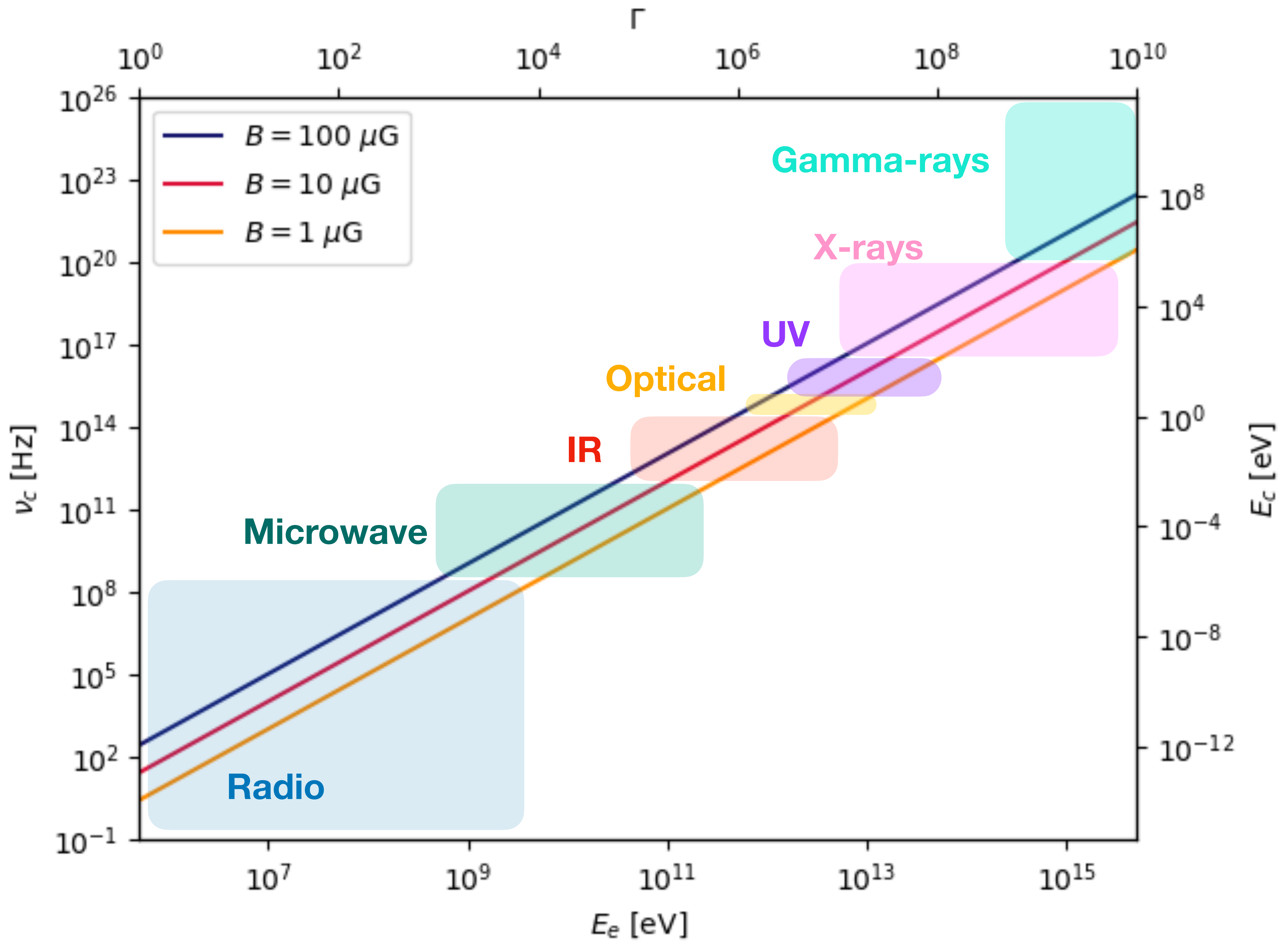


Synchrotron emission

Where is it important?

$$\nu_c = \frac{3}{4\pi}\Gamma^2 \frac{qB\sin\theta}{mc} = \frac{3}{2}\nu_{ch} \equiv \Gamma^2 \nu_L$$

$$\nu_c \simeq 280\ \Gamma^2 \left(\frac{B}{10^{-4}\ \mathrm{G}}\right) Hz$$



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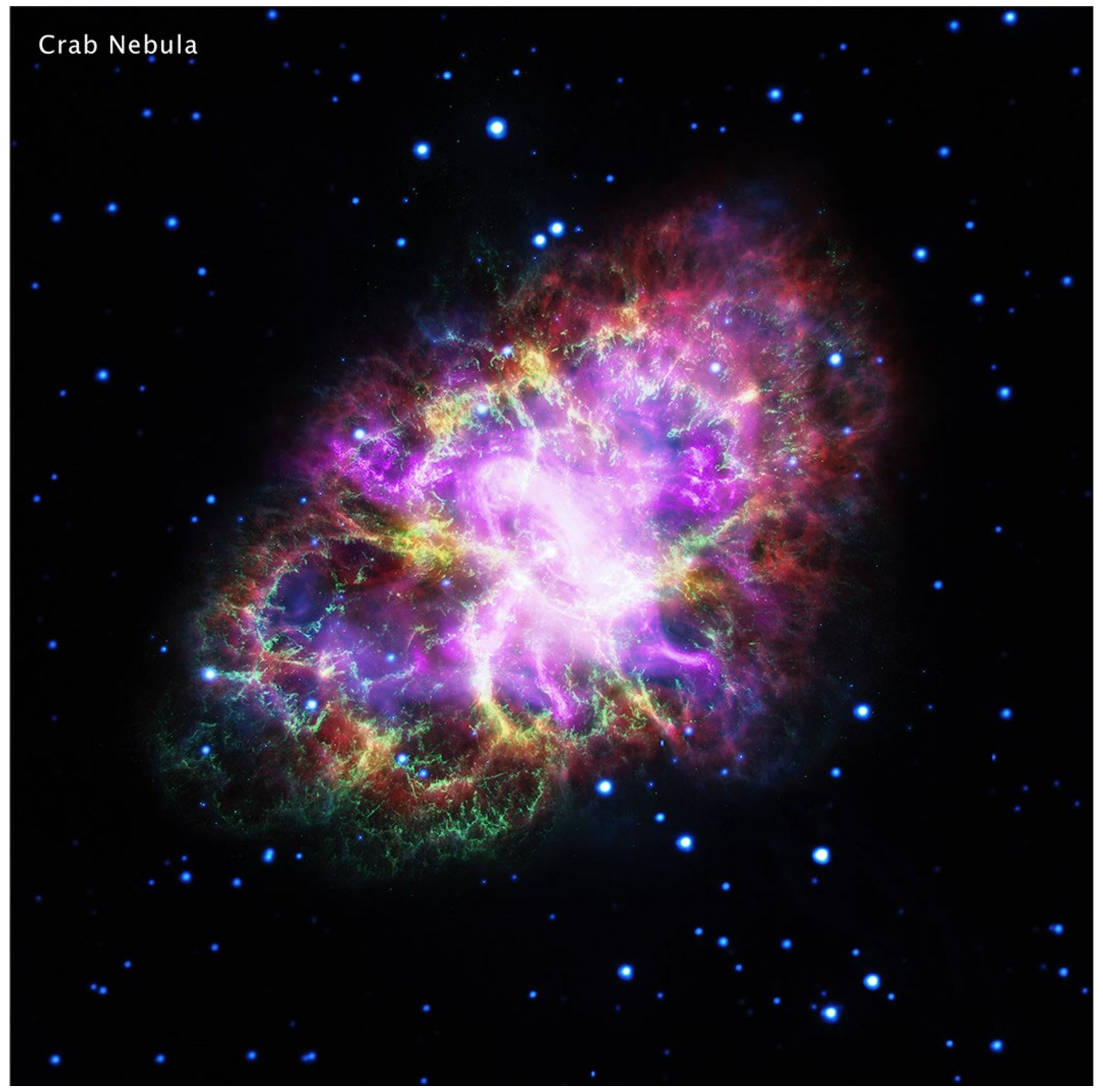
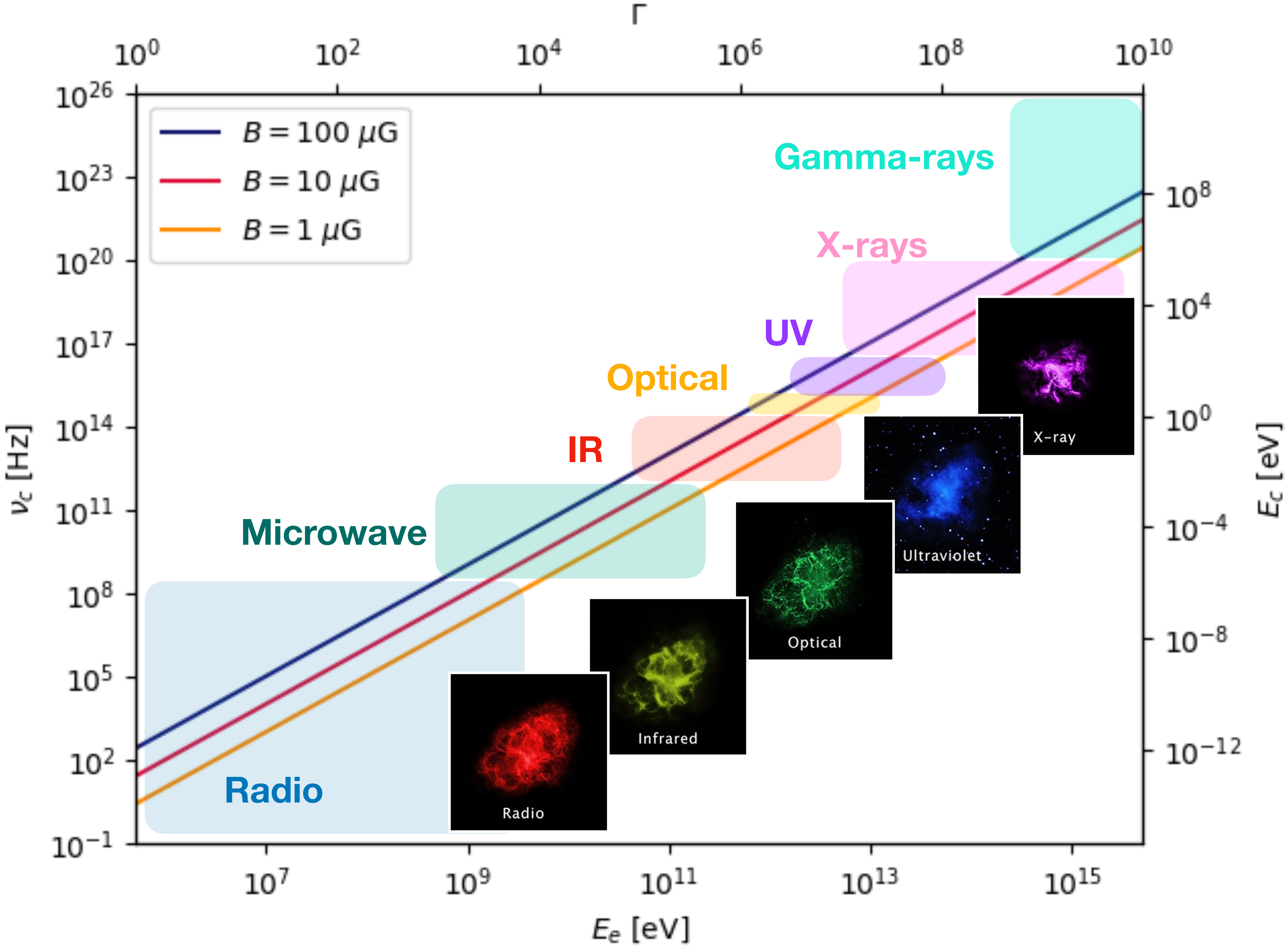


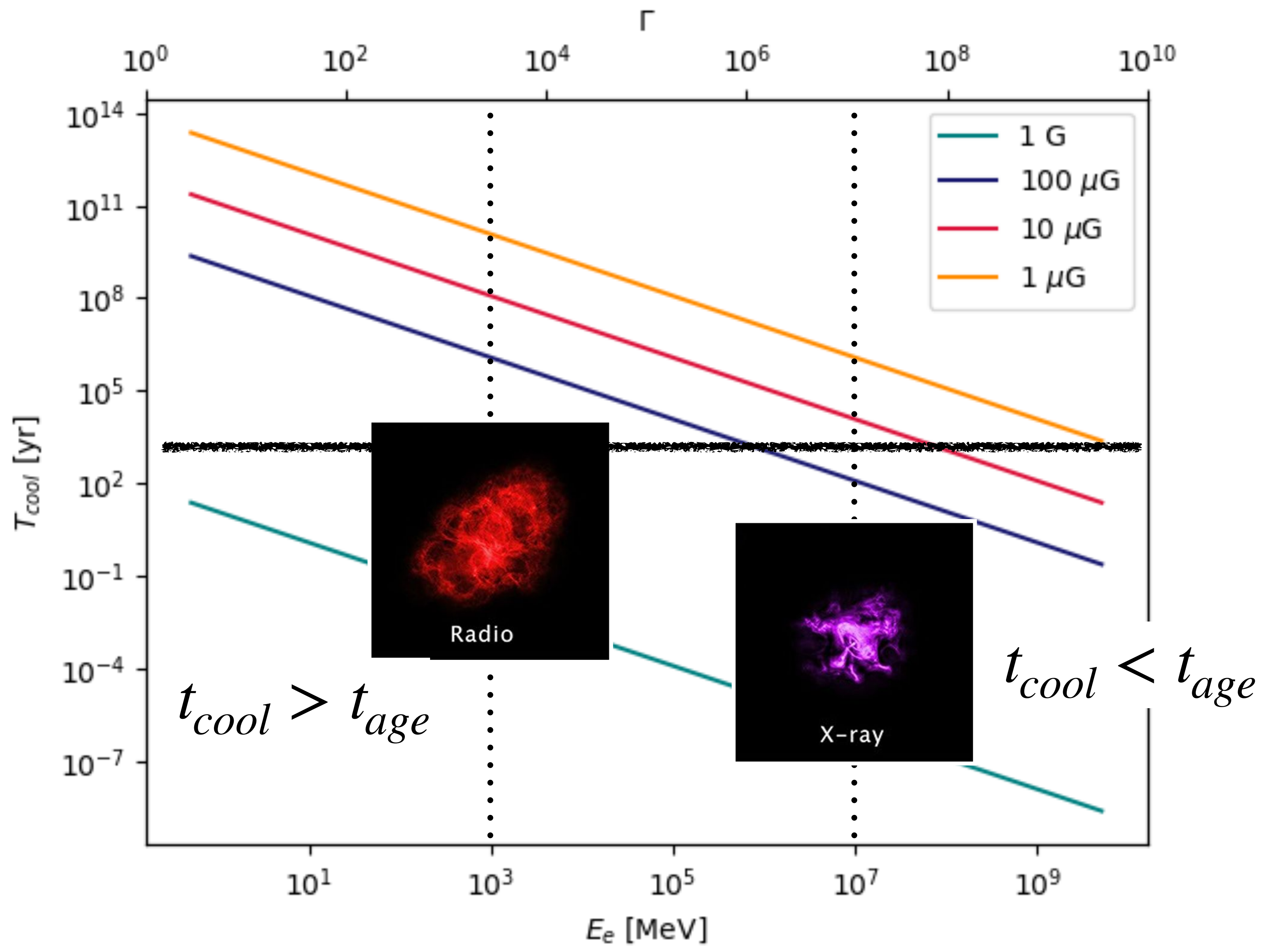


Synchrotron emission

Synchrotron cooling

$$\langle P_{sync} \rangle = \frac{4}{3} c\sigma_T \left(\frac{m_e}{m}\right)^2 \Gamma^2 u_B$$

$$t_{cool}(\Gamma) = \frac{E}{|dE/dt|} = \frac{\Gamma m_e c^2}{\langle P_{sync} \rangle} \propto \Gamma^{-1} = 7.8 \times 10^8 \frac{1}{B^2 \Gamma} \text{ s}$$



The more energetic the electron the faster it cools;

Cooling is faster in a larger magnetic field;

Important mechanism to bend high energy electrons, shaping the emission at high energy!

Synchrotron cooling determine the maximum acceleration power and the morphology of the emission

Synchrotron emission

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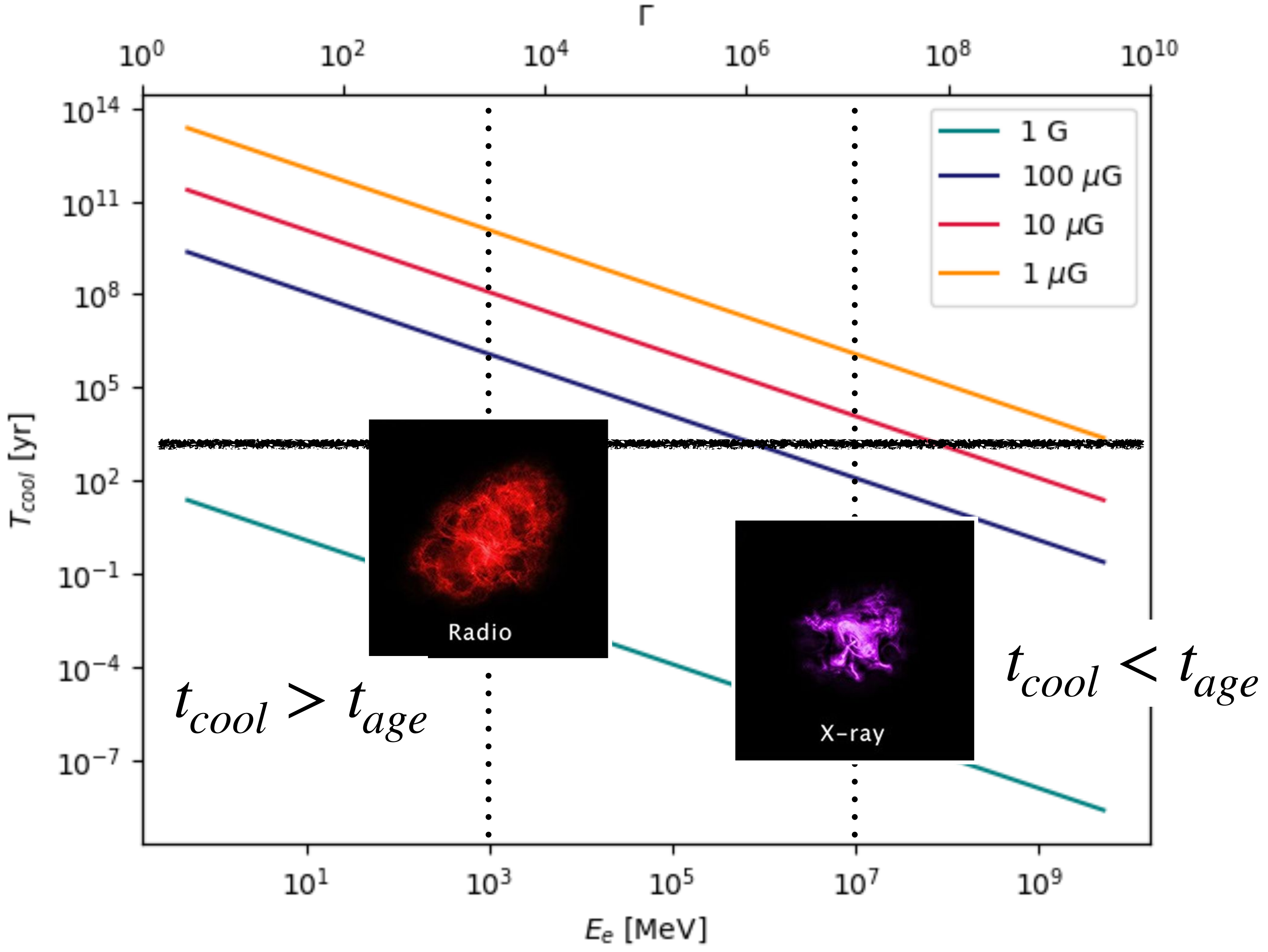

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*True for advection otherwise must be convolved with energy-dependent diffusion: e.g. $D_{Bohm} \propto \Gamma$ therefore electrons cool as fast as they travel!

Synchrotron emission

Synchrotron cooling

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$$-\frac{dE_e}{dt} = \eta(E_e) \propto \Gamma^2 = kE_e^2$$

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Evolution of electrons

Energy losses Injection

$$\frac{\partial N_e(E_e, t)}{\partial t} = \ldots = \frac{\partial}{\partial E}[kE_e^2 N_e(E_e, t)] + Q(E_e, t) \rightarrow kE_e^2 N_e(E_e) = \int Q(E_e) dE_e$$

Missing algebra

$$\rightarrow N_e(E_e) \propto E_e^{-p+1} E_e^{-2}$$

$$E_e^{-p} \rightarrow E_e^{-p-1}$$

Bremsstrahlung

When the braking is given by nuclei

Heuristic approach following Ghisellini (2012)

An electron passing near a nucleus feels its Coulomb field when it is close to it.

We can define an impact parameter ***b*** (minimum radius for interaction) : intuitively related to the number of protons: $N_p \sim b^3 n_p$ where n_p the density of protons

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Then the characteristic time for interaction is $\tau = \dfrac{b}{v} \implies \omega = \dfrac{v}{b}$

The acceleration is given by the Coulomb field

$$a \approx \frac{e^2}{m_e b^2} \quad \Rightarrow \quad P = \frac{2}{3}\frac{q^2}{c^3}|\dot{v}|^2 \propto \frac{e^6}{m_e c^3 b^4}$$

The emitted energy in a single collision is then

$$dW = Pdt \sim \frac{P}{\omega} = \frac{e^6}{m_e c^3 b^3 v}$$

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Easy part: one electron deviated by one proton

Complicated part: considering all electrons and all nuclei (complete derivation in Rybicki & Lightman (1979) and Blumental & Gould (1970))

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Gaunt factor: encloses all electron ions interaction

Thermal ➡ Thermal distribution of electrons with temperature T $j(\nu) = 5.4 \times 10^{-39} Z^2 n_e n_i T^{-1/2} e^{-\frac{h\nu}{kT}} \bar{g}$

Relativistic ➡ Power law distribution of electrons

Bremsstrahlung

When the braking is given by nuclei

Thermal

$$j(\nu) = 5.4 \times 10^{-39} Z^2 n_e n_i T^{-1/2} e^{-\frac{h\nu}{kT}} \bar{g}$$

Relativistic [Haug+1975]

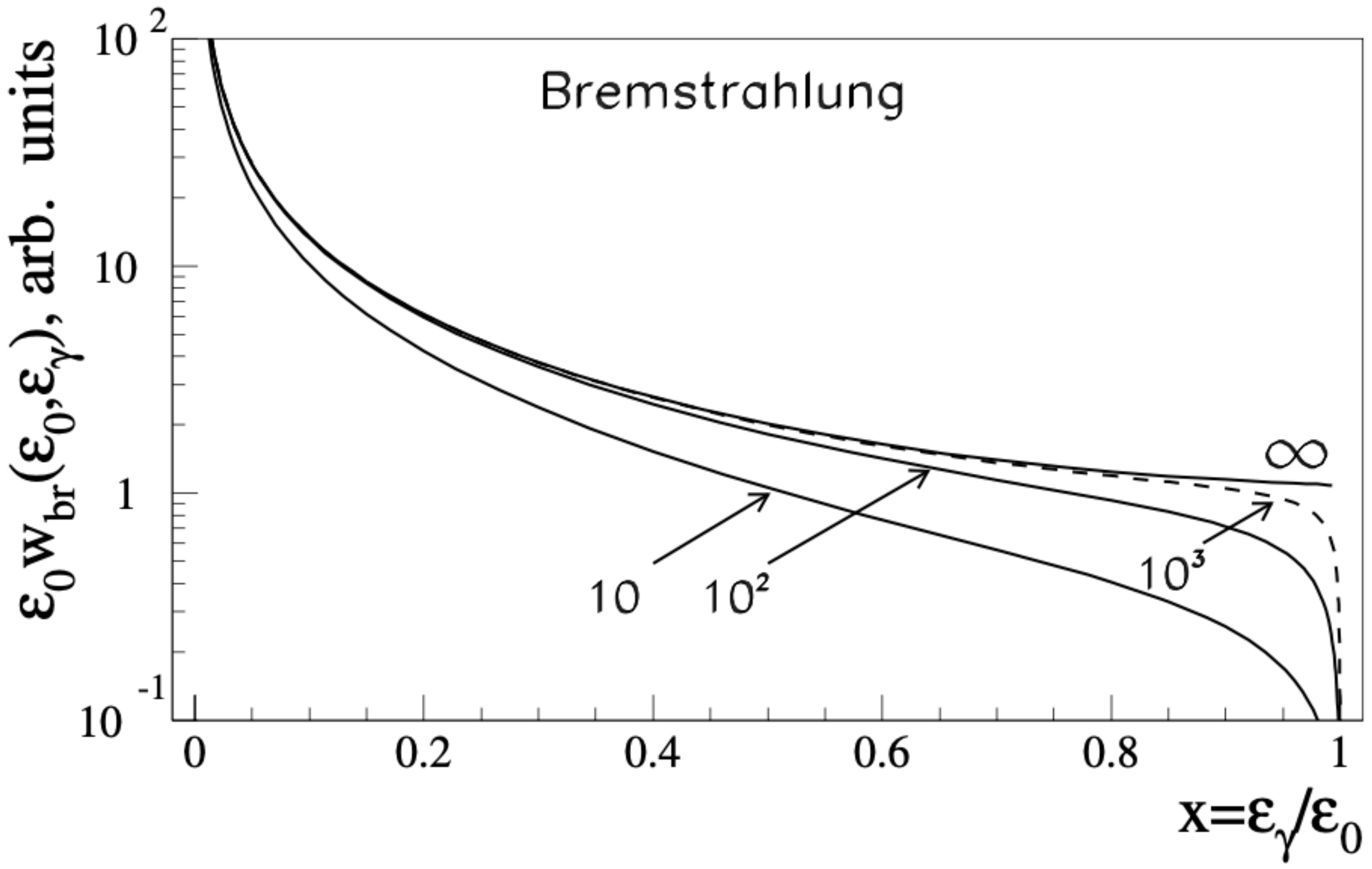
For each electron

$$\frac{dn_\gamma}{dt} = v[(n_p + 4n_{He})\sigma_{e-p}(E_e, E_\gamma) + n_e \sigma_{e-e}(E_e, E_\gamma))]$$

Bethe-Heitler

$$\frac{d\sigma}{dE_\gamma} = \frac{4\alpha r_e^2}{E_\gamma}\left[\left(1 + \frac{(E_e - E_{\gamma^2})}{E^2} - \frac{2}{3}\frac{(E_e - E_\gamma)}{E}\right)\ln\left(\frac{183}{Z^{1/3}}\right) = \frac{1}{9}\frac{E_e - E_\gamma}{E_e}\right]$$

Differential cross section



$$\begin{aligned}
\frac{\sqrt{\varrho^2-4}}{\pi}\int A\,\mathrm{d}\Omega_{p_1'} = \sqrt{\varrho^2-4}\,\Big\{ & \frac{w^2+\varrho^2}{4x_1x_2}\left(\frac{x_1-x_2}{x}\right)^2 - \frac{1}{4}\left(\frac{1}{x_1}-\frac{1}{x_2}\right)^2 - \frac{\varrho^2}{2x^2} + \frac{2\varrho^2}{(w^2-4)x_1x_2}\left(1+\frac{1}{w^2-4}\right) \\
& + \frac{4\varrho^2}{w^2(w^2-4)x_1^4}\Big[3\frac{(w^2-2)^2}{w^2-4}x_1x_2 - 2x^2\Big\{1+\frac{6}{w^2(w^2-4)}\Big\}\Big] - \frac{4\varrho^2 x}{(w^2-4)x_1^3} \\
& + \frac{\varrho^2}{x_1^2}\Big[\frac{4}{w^2} - \frac{3}{2} - \frac{8}{w^2(w^2-4)^2} + \frac{(w^2-2)x}{w^2(w^2-4)}\Big] - \frac{\varrho^2}{(w^2-4)x_1} + \frac{1}{R_1}\Big[\frac{w^2-4x_2}{\varrho^2} - \frac{w^2(w^2-4)}{4x_1x_2} - \frac{4}{x_1} \\
& + \frac{1}{x_1^2}\Big(w^2 - \frac{w^2-2}{2}\varrho^2\Big)\Big]\Big\} + \frac{L}{\sqrt{R_1}}\Big\{4 + \frac{1}{x_2}\Big(10 - \frac{w^2}{2}\Big) - \frac{2}{x_1}(2w^2 - x_2 + 4) + \frac{3w^2(w^2-4)}{4x_1x_2} \\
& + \frac{4}{x_1R_2}\Big(x_2 - 3x_1 + 4 + 2\frac{\varrho^2-3}{x_1}\Big) + \frac{w^2-4-4x_1x_2/\varrho^2}{x_1R_1}\Big[\frac{w^2R_2}{4x_2} - (\varrho^2-2) - 2x_1\Big]\Big\} \\
& + \varrho L_1\Big\{\frac{\varrho^2+2}{x^2} + \frac{8}{x_1^2}\Big\} + \frac{L_2}{W_2}\Big\{2\frac{w^2-2}{x_2} - \frac{\varrho^2-2-x_2}{x_1} + \frac{\varrho^2-2}{2x}(\varrho^2+x_1) + \frac{w^2-4}{R_2} \\
& - 2 + \frac{\varrho^2-2}{8xx_1}(w^2+\varrho^2-4)^2 + \frac{1}{R_2x_1}\Big[(\varrho^2-2)\Big\{\Big(\frac{w^2+\varrho^2}{2} - 2\Big)x_2 - (w^2-2)\Big\} - 2x_2^2 - 4x_2\Big] \\
& + \frac{1}{R_2x}\Big[\frac{\varrho^2-2}{2}\{3(\varrho^2-4) - w^2(\varrho^2-5)\} + 2x_1^2 - 6x_1 + (w^2-2)x_2\Big]\Big\} \qquad \text{(A1)} \\
& + \frac{2\varrho L_3}{w\sqrt{w^2-4}\,x_1}\Big\{4 + \frac{8(w^2-2)}{(w^2-4)^2} - \frac{3(w^2-2)}{2x_2} + \frac{1}{x_1}[x_2^2 - 2w^2 - (w^2-1)x_2 + \tfrac{1}{2}(w^2-2)^2] \\
& + \frac{\varrho^2}{2x}(\varrho^2-2) - \frac{x_2}{2x}(w^2-2) - \frac{w^2+\varrho^2-4}{4xx_2}(w^2-2)^2 + \frac{x}{4x_1x_2}\Big[(w^2-2)^2 \\
& + (\varrho^2-2)(\varrho^2-4) - 8\frac{\varrho^2-2}{w^2-4}\Big] + \frac{4x^2}{w^2(w^2-4)x_1} - \frac{1}{R_2}\Big[2(\varrho^2-2)x_2 + (w^2-4)x - 4(w^2-2) \\
& + \frac{8-\varrho^2}{2x_1}(w^2-2)\Big] + \frac{1}{xR_2}\Big[\frac{\varrho^2-2}{2}\{3(\varrho^2-4) - w^2(\varrho^2-5)\} - x_1(w^2-2x_1+4)\Big] \\
& + \frac{2}{(w^2-4)^2x_1}\Big[w^2(w^2-2)(\varrho^2-4) - 2(\varrho^2-2) + 4\frac{\varrho^2}{w^2}\Big] + \frac{4(w^2-2)x}{(w^2-4)x_1^2}\Big[\frac{12x}{w^2(w^2-4)} - \frac{\varrho^2-2}{2}\Big] \\
& \cdot\Big[1 - \frac{x}{w^2x_1}\Big]\Big\} - \frac{L_4}{W_4}\Big\{1 + \frac{\varrho^2-2}{8x_1x_2}\Big[(w^2-2)^2 + (\varrho^2-2)^2 - 6(w^2+\varrho^2-4) + \frac{16x}{w^2-4}\Big] \\
& + \frac{2}{x_1x_2}(1 - x_1 - x_1^2) + \frac{1}{w^2-4}\Big(\varrho^2 - 4 - \frac{8}{w^2-4}\Big)\Big\} + \{x_1 \longleftrightarrow x_2\}.
\end{aligned}$$

Bremsstrahlung

In practice: when it is important

Electrons passing through a plasma lose energy due to bremsstrahlung and emit photons. After a certain length $X_0 = ct_0 \propto (n\sigma_0)^{-1}$ the energy of the electron is reduced by a factor e, therefore after a certain time:

$$-\frac{dE_e}{dt} = \left(\frac{cm_p n}{X_0}\right) E_e \qquad t_{br} = \frac{E_e}{\frac{dE_e}{dt}} \simeq 4 \times 10^7 \left(\frac{n}{\mathrm{cm}^{-3}}\right)^{-1} \mathrm{yr}$$

Independent of E_e !

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Independent of E_e !

If parent electrons follow a PL distribution:

$$N_e(E_e) \propto \underbrace{\left(\frac{dE_e}{dt}\right)^{-1}}_{\propto E^{-1}} \underbrace{\int_{E_e} Q(E_e) dE_e}_{\propto E^{-p+1}}$$

- Bremsstrahlung do not change power law electrum spectrum
- The spectrum of bremsstrahlung follows the spectrum of parent electrons!

Bremsstrahlung

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Compare with:

$$t_{pp} = (n\sigma_{PP}c)^{-1} \simeq 5.4 \times 10^7 \left(\frac{n}{\mathrm{cm}^{-3}}\right)^{-1} \mathrm{yr}$$

Bremsstrahlung important only when Electrons >> Protons

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Bremsstrahlung important only when Electrons >> Protons

But: pp interactions have a threshold, below the threshold bremsstrahlung dominates (Unfortunately it is in the MeV domain!)

(Inverse) Compton scattering

We know that radiation is emitted if electric charge are accelerated. An incoming electromagnetic wave (a photon) makes the electron oscillate in response to the electric force. The electron then in turn emits a photon of energy $h\nu_f \lesssim h\nu_i$ (due to recoil)

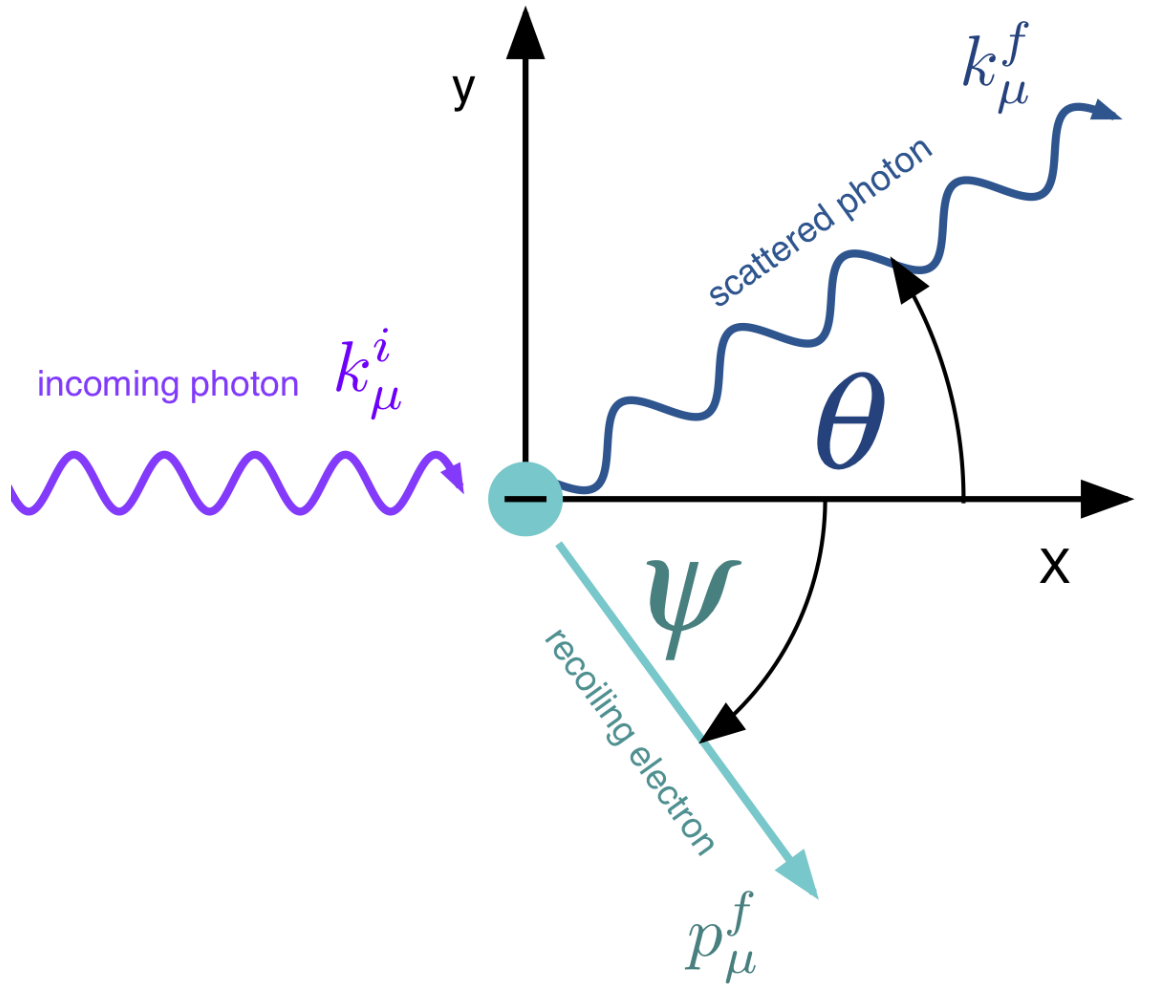
If $h\nu \ll m_e c^2$: **Thomson scattering** (classic treatment)

If $h\nu \gtrsim m_e c^2$: **Compton scattering** (quantistic treatment)

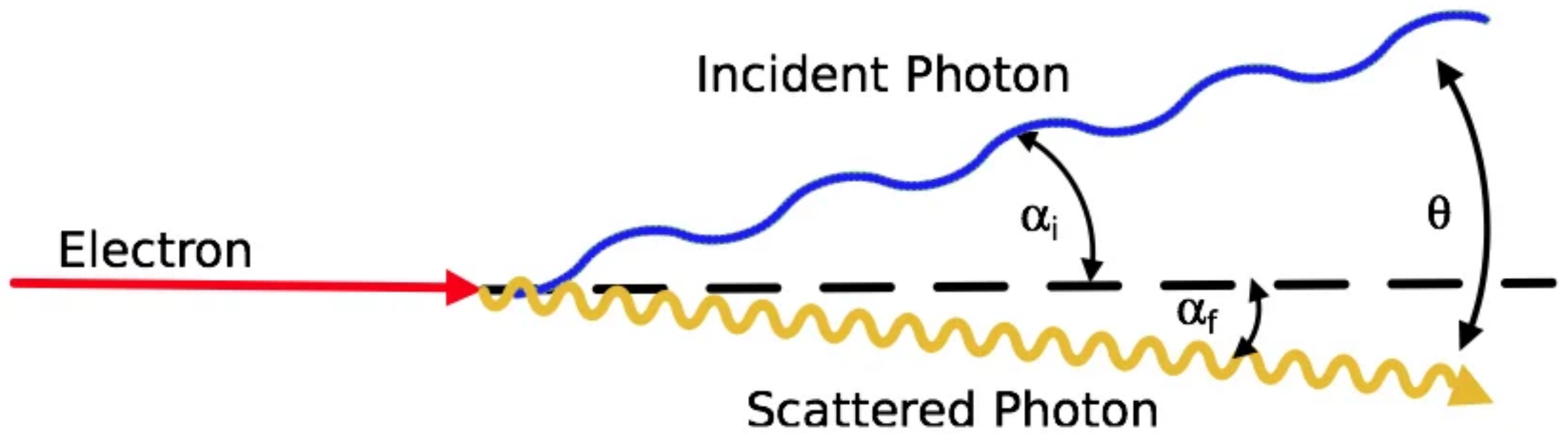
Considering the collision in the quantistic treatment, we can imagine also the inverse effect: a low energy photon, impacting on a relativistic electrons and *getting energy from it*: **Inverse Compton scattering.**

Among the main emission mechanism to produce high energy photons.

Direct Compton



Inverse Compton



Compton scattering

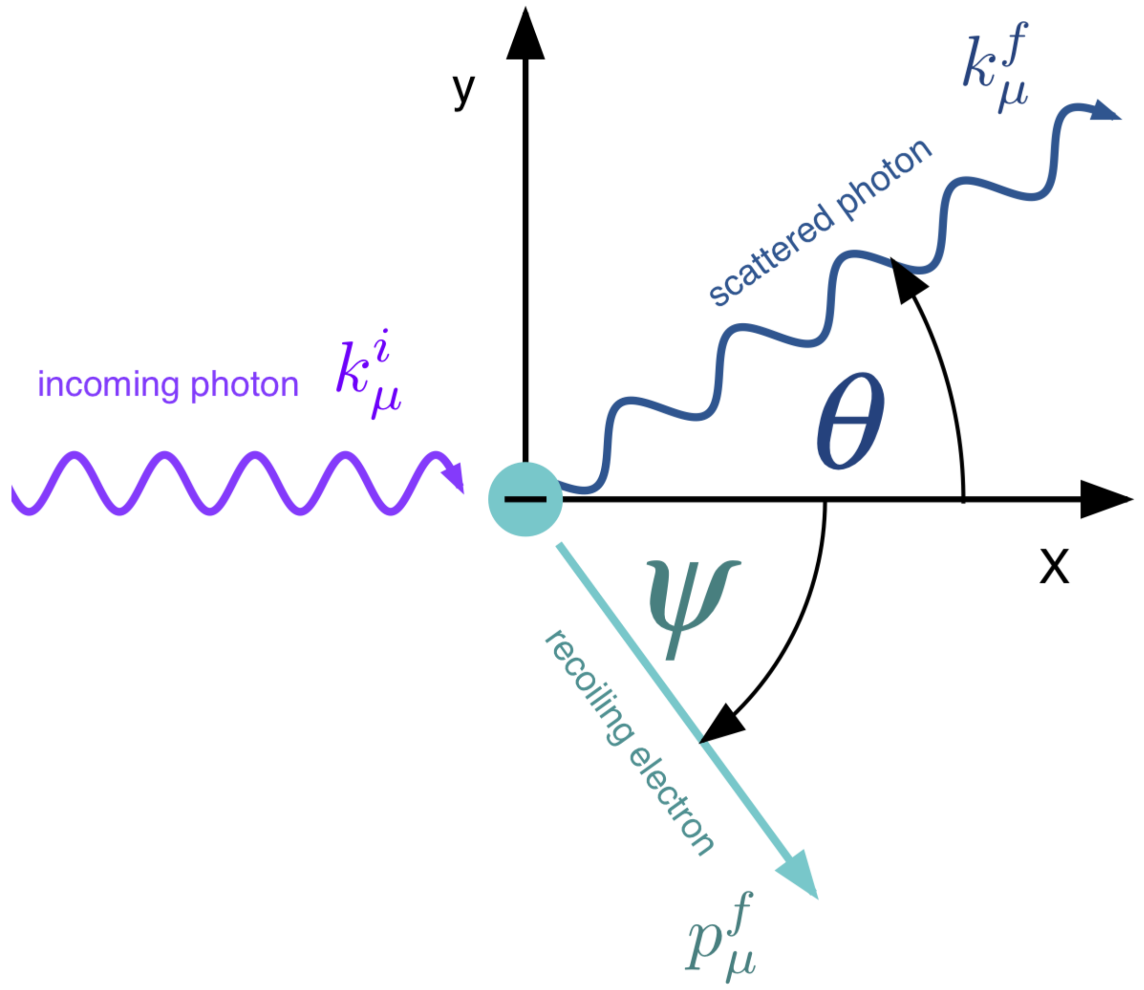
Energy exchange

We compute the change of energy in the case of direct Compton scattering: this will help us evaluating the energy exchange in the Inverse Compton (via relativistic transformations)

4-momenta

	Before	After	
	$\tilde{P}_i = \dfrac{h\nu_i}{c}(1,\vec{n}_i),$	$\tilde{P}_f = \dfrac{h\nu_f}{c}(1,\vec{n}_f)$	Photon
	$\tilde{Q}_i = m_e(c,0),$	$\tilde{Q}_f = \Gamma_f m_e(c,\vec{v}_f)$	Electron

$$\tilde{P}_i + \tilde{Q}_i = \tilde{P}_f + \tilde{Q}_f$$



Compton scattering

Energy exchange

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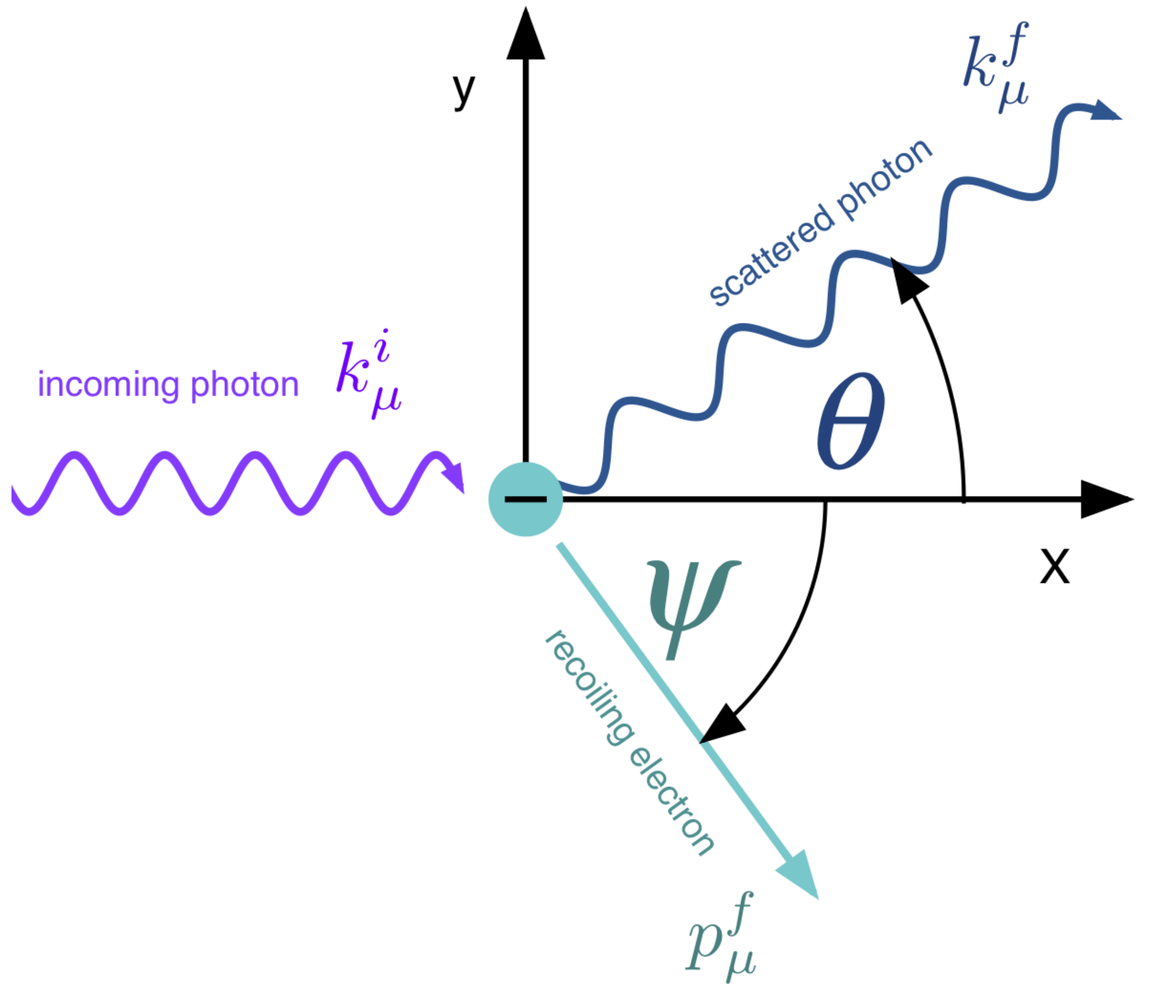

$$\tilde{P}_i + \tilde{Q}_i = \tilde{P}_f + \tilde{Q}_f$$

$$\Rightarrow \tilde{Q}_f^2 = (\tilde{P}_i + \tilde{Q}_i - \tilde{P}_f)^2$$

$$\tilde{Q}_f^2 = \tilde{P}_i^2 + \tilde{Q}_i^2 + \tilde{P}_f^2 + 2\tilde{P}_i\tilde{Q}_i - 2\tilde{P}_i\tilde{P}_f - 2\tilde{Q}_i\tilde{P}_f$$

$$\tilde{P}^2 = 0$$

$$\tilde{Q}^2 = \Gamma^2 m_e^2 c^2 - \Gamma^2 m_e^2 v^2 = m_e^2 c^2$$

$$\tilde{P}_i\tilde{P}_f = \tilde{Q}_i(\tilde{P}_i - \tilde{P}_f)$$

$$h\nu_i h\nu_f(1 - \vec{n}_i \cdot \vec{n}_f) = m_e(h\nu_i - h\nu_f)c^2$$

$$h\nu_i\nu_f(1 - \cos\theta) = m_e c^2(\nu_i - \nu_f)$$

$$\nu_f = \frac{\nu_i}{1 + \frac{h\nu_i}{m_e c^2}(1 - \cos\theta)}$$

Compton scattering

Energy exchange

$$\nu_f = \frac{\nu_i}{1 + \frac{h\nu_i}{m_e c^2}(1 - \cos\theta)}$$

$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta) = \lambda_c(1 - \cos\theta) \geq 0$$

Compton wavelength
~ 2.426×10^{-10} cm

If $h\nu_i \ll m_e c^2$:
Thomson limit

$\epsilon_i \sim \epsilon_f$
The scattering can be considered elastic

$$\epsilon_f = \frac{\epsilon_i}{1 + \epsilon_i(1 - \cos\theta)}$$

If $h\nu_i \ll m_e c^2$ we can do a Taylor expansion

$$\frac{\Delta\epsilon}{\epsilon_i} = \frac{\nu_f - \nu_i}{\nu_i} \simeq \frac{1}{\nu_i}\left[\nu_i\left(1 - \frac{h\nu_i}{m_e c^2}[1 - \cos\theta]\right) - \nu_i\right] = -\frac{h\nu_i}{m_e c^2}(1 - \langle\cos\theta\rangle) = -\frac{h\nu_i}{m_e c^2}$$

If $h\nu_i > m_e c^2$: need **Klein-Nishina** treatment as the scattering is inelastic

Compton scattering

Energy exchange

$$\nu_f = \frac{\nu_i}{1 + \frac{h\nu_i}{m_e c^2}(1 - \cos\theta)}$$

$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta) = \lambda_c(1 - \cos\theta) \geq 0$$

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If $h\nu_i \ll m_e c^2$ we can do a Taylor expansion

The photon loses a small amount of energy

$$\frac{\Delta\epsilon}{\epsilon_i} = \frac{\nu_f - \nu_i}{\nu_i} \simeq \frac{1}{\nu_i}\left[\nu_i\left(1 - \frac{h\nu_i}{m_e c^2}[1 - \cos\theta]\right) - \nu_i\right] = -\frac{h\nu_i}{m_e c^2}(1 - \langle\cos\theta\rangle) = -\frac{h\nu_i}{m_e c^2}$$

If $h\nu_i > m_e c^2$: need **Klein-Nishina** treatment as the scattering is inelastic

Inverse Compton scattering

Energy exchange

In the Inverse Compton the electron moves with velocity v and the energy can be transferred from the electron to the photon.

Trick: we can use the expression derived for the direct Compton scattering in the electron rest frame K' and transform using Lorentz transformations

Direct Compton

$$K: \quad \epsilon_f = \frac{\epsilon_i}{1 + \frac{\epsilon_i}{m_e c^2}(1 - \cos\alpha)}$$

$$K': \quad \epsilon'_f = \frac{\epsilon'_i}{1 + \frac{\epsilon'_i}{m_e c^2}(1 - \cos\alpha')}$$

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* **α is the scattering angle**, hence the angle between the initial and final photon direction : $\cos\alpha' = \overrightarrow{n'_f} \cdot \overrightarrow{n'_i}$

In spherical coordinates: $(\vec{n}'_i) : (\sin\theta'_i \cos\phi'_i, \sin\theta'_i \sin\phi'_i, \cos\theta'_i)$
$(\vec{n}'_f) : (\sin\theta'_f \cos\phi'_f, \sin\theta'_f \sin\phi'_f, \cos\theta'_f)$

$$\cos\alpha' = \cos\theta'_i \cos\theta'_f + \sin\theta'_i \sin\theta'_f \cos(\phi'_i - \phi'_f)$$

Relativistic aberration $\quad \cos\theta = \dfrac{\cos\theta' + \beta}{1 + \beta\cos\theta'}$

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$$K': \quad \epsilon'_f = \frac{\epsilon'_i}{1 + \frac{\epsilon'_i}{m_e c^2}(1 - \cos\alpha')}$$

Lorentz transformations between K' and K

$$\epsilon'_i = \epsilon_i \Gamma(1 - \beta\cos\theta_i)$$

$$\epsilon_f = \frac{\epsilon'_f}{\Gamma(1 - \beta\cos\theta_f)} = \epsilon'_f \Gamma(1 + \beta\cos\theta'_f)$$

$\epsilon'_f = \dfrac{\epsilon'_i}{1 + \epsilon'_i(1 - \cos\alpha')}$ $\quad \epsilon'_i = \epsilon_i\Gamma(1 - \beta\cos\theta_i)$

$$\epsilon_f = \frac{\epsilon_i \Gamma^2 (1 - \beta\cos\theta_i)(1 + \beta\cos\theta'_f)}{1 + \frac{\epsilon_i}{m_e c^2}\Gamma(1 - \beta\cos\theta_i)(1 - \cos\alpha')}$$

Inverse Compton scattering

Energy exchange

$$\epsilon_f = \frac{\epsilon_i \Gamma^2 (1 - \beta \cos\theta_i)(1 + \beta \cos\theta'_f)}{1 + \epsilon_i \Gamma (1 - \beta \cos\theta_i)(1 - \cos\alpha')}$$

In Thomson regime $\epsilon'_f \simeq \epsilon'_i \iff 1 - \cos\alpha' = 0$:

$$\epsilon'_i \ll m_e c^2 \iff \epsilon_i \ll \frac{m_e c^2}{\Gamma}$$

$$\epsilon_f = \frac{\epsilon_i \Gamma^2 (1 - \beta \cos\theta_i)(1 + \beta \cos\theta'_f)}{1 + \cancel{\epsilon_i \Gamma (1 - \beta \cos\theta_i)(1 - \cos\alpha')}} \boxed{\propto \Gamma^2 \epsilon_i}$$

Head on scattering $\theta_i = \pi$:

$$\epsilon_f = \epsilon_i \Gamma^2 (1 + \beta)(1 + \beta \cos\theta'_f)$$

Maximize energy if $\theta'_f = 0$:

$$\epsilon_f^{max} = \epsilon_i \Gamma^2 (1 + \beta)(1 + \beta) \sim 4\Gamma^2 \epsilon_i$$
$$= \epsilon_i \frac{1}{1 - \beta^2} (1 + \beta)^2 = \epsilon_i \frac{1 + \beta}{1 - \beta}$$

Inverse Compton scattering

Energy exchange

$$\epsilon_f = \frac{\epsilon_i \Gamma^2 (1 - \beta \cos\theta_i)(1 + \beta \cos\theta_f')}{1 + \frac{\epsilon_i}{m_e c^2}\Gamma(1 - \beta\cos\theta_i)(1 - \cos\alpha')}$$

In the Klein-Nishina limit:

$$\epsilon_i' \gg m_e c^2 \implies \epsilon_i \gg \frac{m_e c^2}{\Gamma}$$

$$\approx \frac{\cancel{\epsilon_i}\Gamma^2 \cancel{(1 - \beta \cos\theta_i)}(1 + \beta\cos\theta_f')}{\frac{\cancel{\epsilon_i}}{m_e c^2}\Gamma\cancel{(1 - \beta\cos\theta_i)}(1 - \cos\alpha')} \propto \boxed{\Gamma m_e c^2}$$

Inverse Compton scattering

Energy exchange

$$\epsilon_f = \frac{\epsilon_i \Gamma^2 (1 - \beta \cos\theta_i)(1 + \beta \cos\theta_f')}{1 + \frac{\epsilon_i}{m_e c^2}\Gamma(1 - \beta\cos\theta_i)(1 - \cos\alpha')}$$

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Summary:

Condition for Thomson limit:

$$\epsilon_i' \ll m_e c^2 \implies \epsilon_i \ll \frac{m_e c^2}{\Gamma}$$

$$\epsilon_f \propto \Gamma^2 \epsilon_i$$

In Thomson photons are boosted by Γ^2

Klein-Nishina limit if:

$$\epsilon_i' \gg m_e c^2 \implies \epsilon_i \gg \frac{m_e c^2}{\Gamma}$$

$$\epsilon_f \propto \Gamma m_e c^2$$

In KN photons are emitted at energies similar to the electron

The photon always gain energy; in the KN limit a large fraction of energy of electron is transferred

Inverse Compton scattering

Thomson and Klein-Nishina limits: where do they apply

$$\epsilon_f = \frac{\epsilon_f'}{\Gamma(1-\beta\cos\theta_f)} = \epsilon_f'\Gamma(1+\beta\cos\theta_f')$$

Condition for Thomson regime:

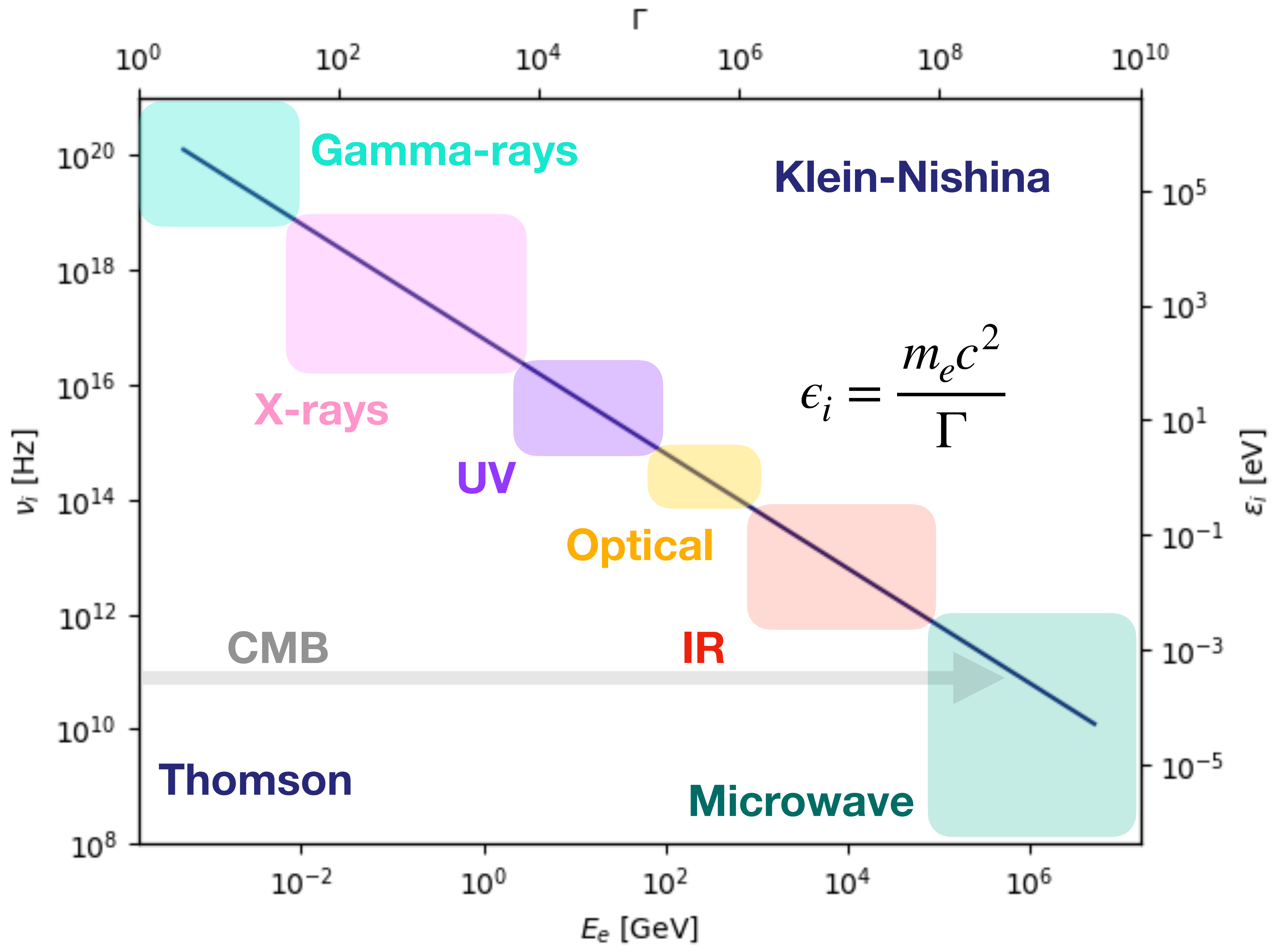
$$\epsilon_i' \ll m_e c^2 \implies \epsilon_i \ll \frac{m_e c^2}{\Gamma} \qquad \epsilon_f \propto \Gamma^2 \epsilon_i$$

Evaluated in the system where the electron is at rest

In the lab. system

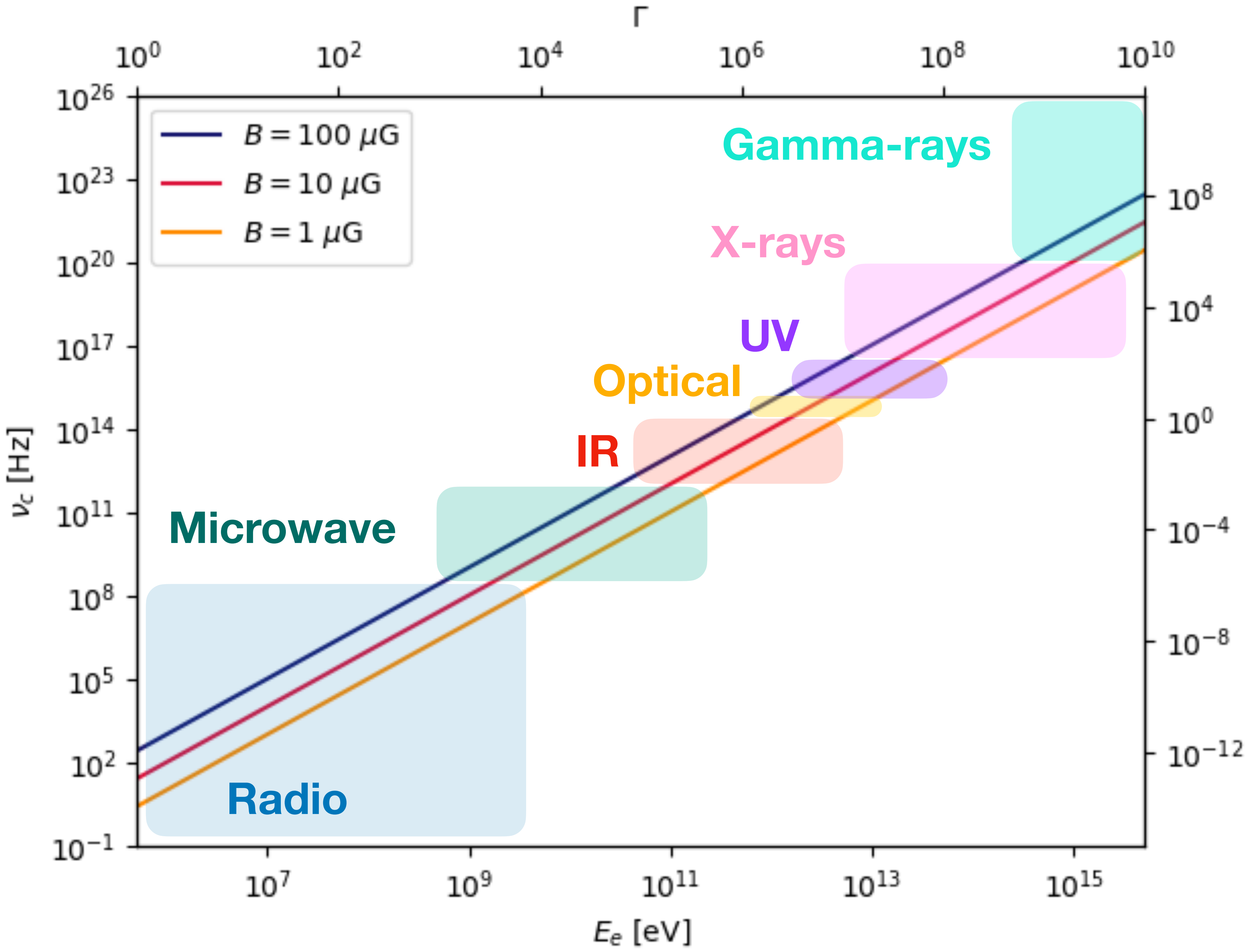
Klein-Nishina limit if:

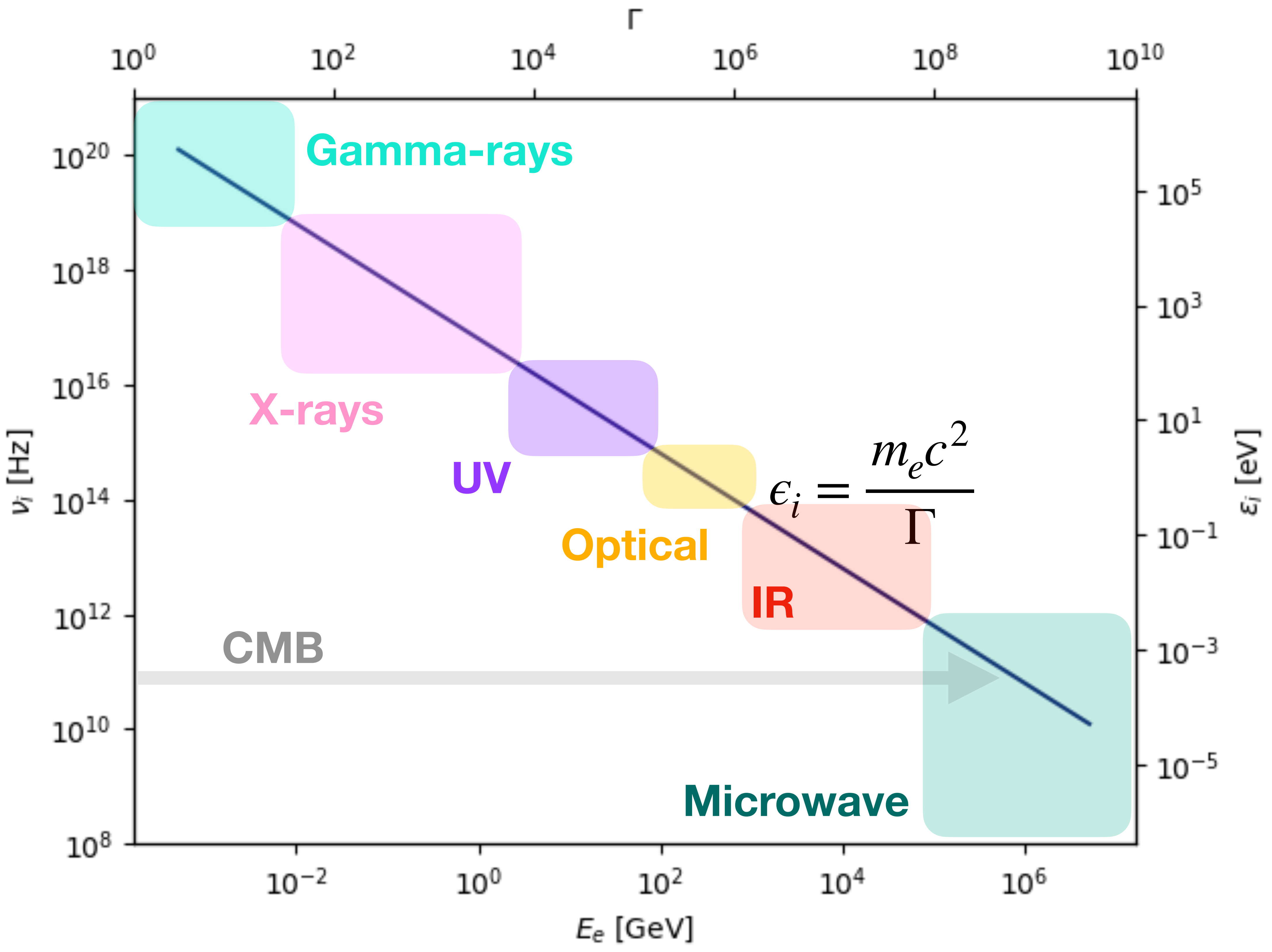
$$\epsilon_i' \gg m_e c^2 \implies \epsilon_i \gg \frac{m_e c^2}{\Gamma} \qquad \epsilon_f \propto \Gamma m_e c^2$$



Inverse Compton scattering

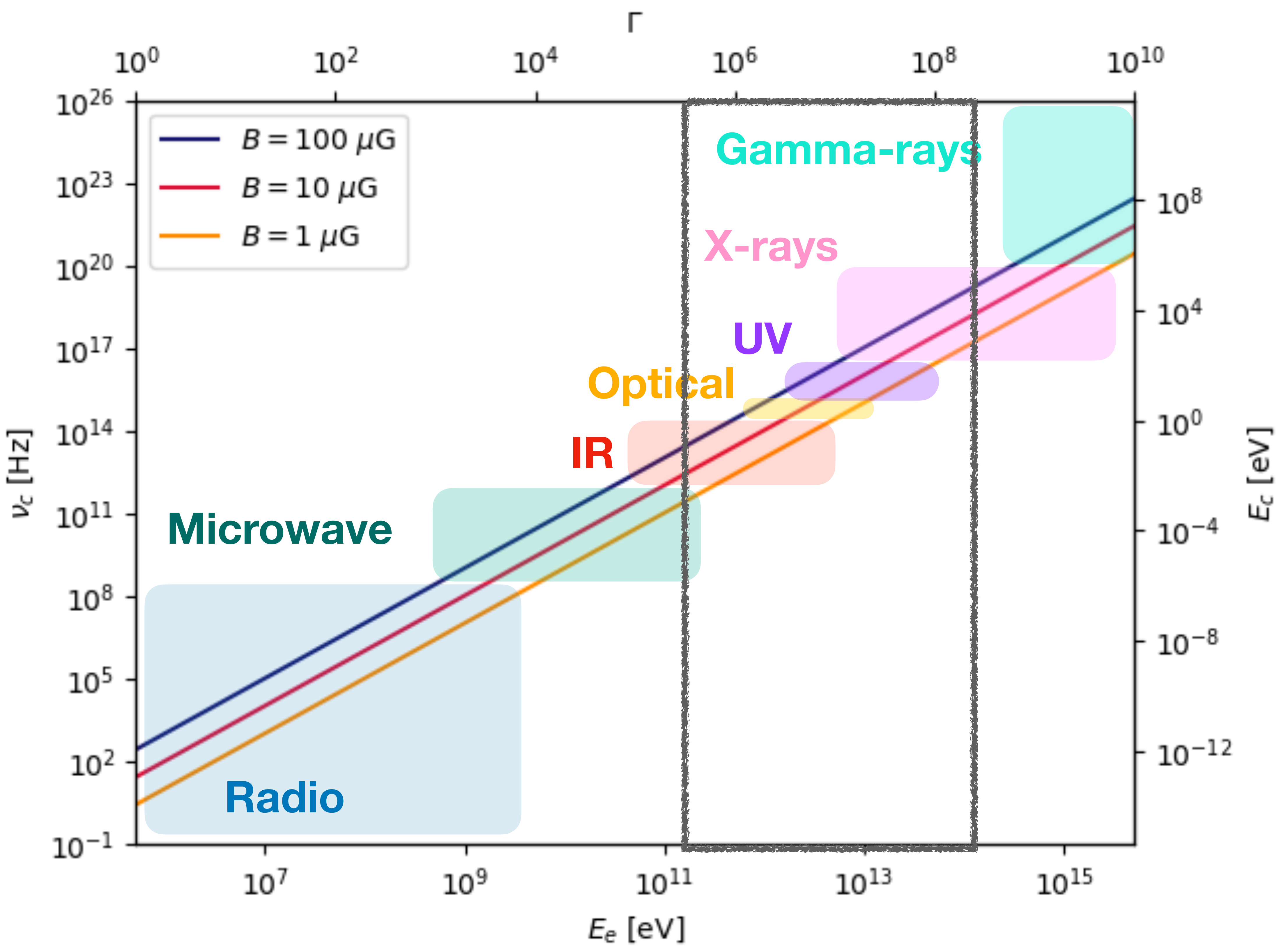
Where do high energy electron emits?





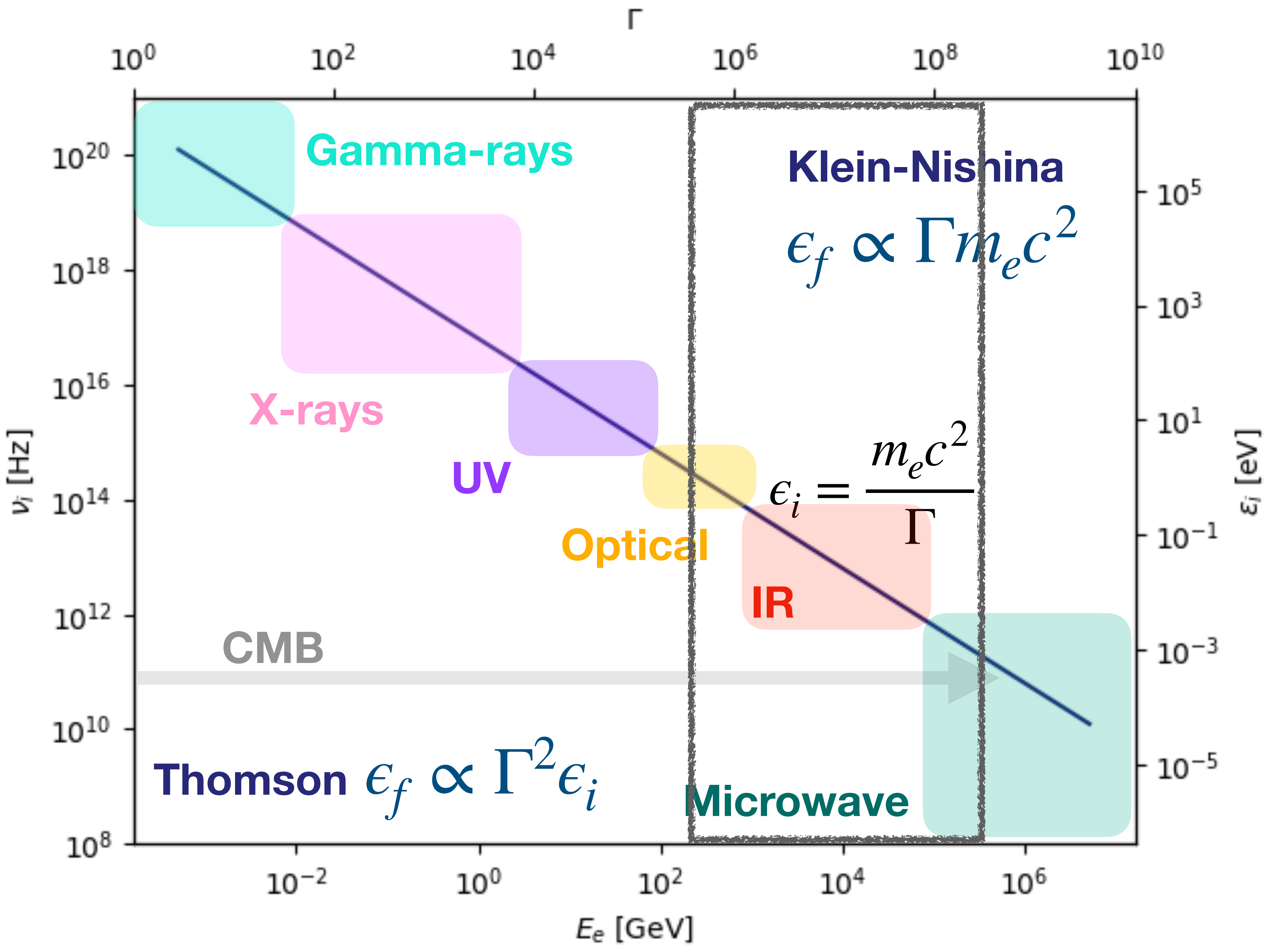
Inverse Compton scattering

Where do high energy electron emits?



100 GeV -100 TeV

Characteristic Synchrotron emission is in the Optical-UV, and soft X



100 GeV -100 TeV

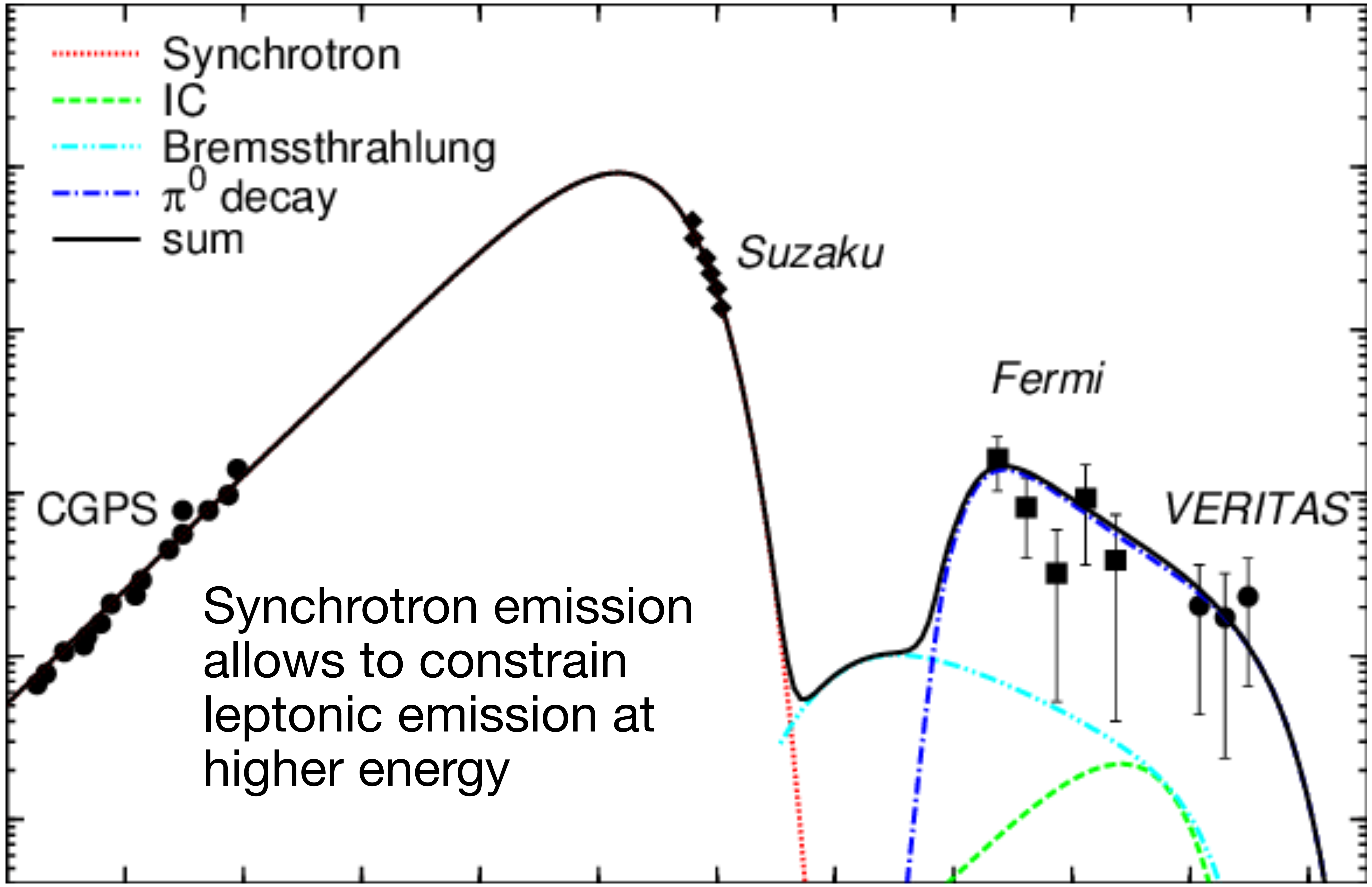
Klein-Nishina is triggered by IR photons. $\epsilon_f \sim E_e$

CMB triggers KN for Ee >100 TeV

CTAO probed energies

Inverse Compton vs. Synchrotron

The example of young SNR Tycho (Morlino & Caprioli 2012)



Inverse Compton scattering

Emitted power

Larmor formula $P = \frac{2}{3}\frac{q^2}{c^3}|\dot{\vec{v}}|^2$

$$|\dot{v}|^2 \sim \left(\frac{eE}{m}\right)^2$$

$$S = cU_{rad} = \frac{c}{4\pi}E^2$$

$$P = \frac{2}{3}\frac{e^2}{c^3}\frac{e^2}{m^2}4\pi U_{rad} = \sigma_T c U_{rad}$$

This is in the Thomson limit

Trick: *we evaluate the emitted power in the Thomson limit where the electron is at rest*

Emitted power via Larmor

Evaluated in the system where the electron is at rest

$$-\frac{dE'}{dt'} = \sigma_T c U'_{rad} = \sigma_T c \int N'_{ph}(\epsilon')\epsilon'_f(\epsilon', \alpha')d\epsilon'$$

Use Lorentz invariant: $\frac{N_{ph}(\epsilon)d\epsilon}{\epsilon} = \frac{N'_{ph}(\epsilon')d\epsilon'}{\epsilon'}$ ***and*** $\epsilon' = \epsilon\Gamma(1 - \beta\cos\theta)$

$$U'_{rad} = \int \epsilon' N'_{ph}(\epsilon')d\epsilon' = \int \epsilon'^2 \frac{N_{ph}(\epsilon)d\epsilon}{\epsilon} = \int \epsilon^2\Gamma^2(1-\beta\cos\theta)^2\frac{N_{ph}(\epsilon)d\epsilon}{\epsilon} = \int \epsilon^{\cancel{2}}\Gamma^2(1-\beta\cos\theta)^2\frac{N_{ph}(\epsilon)d\epsilon}{\cancel{\epsilon}} = \ldots = \Gamma^2\left(1+\frac{\beta^2}{3}\right)U_{rad}$$

Averaging over the cosine

Inverse Compton scattering

Emitted power

Larmor formula $P = \frac{2}{3}\frac{q^2}{c^3}|\dot{\vec{v}}|^2$

$$|\dot{v}|^2 \sim \left(\frac{eE}{m}\right)^2$$

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$$-\frac{dE'}{dt'} = \sigma_T c U'_{rad} = \sigma_T c \int N'_{ph}(\epsilon')\epsilon'_f(\epsilon', \alpha')d\epsilon'$$

$$= \sigma_T c \Gamma^2\left(1 + \frac{\beta^2}{3}\right)U_{rad}$$ This is the *emitted* power…

… electrons also *absorb* energy from the photons.

Inverse Compton scattering

Energy losses

$$-\frac{dE'}{dt'} = \sigma_T c \Gamma^2 \left(1 + \frac{\beta^2}{3}\right) U_{rad}$$ This is the *emitted* power…

… electrons also *absorb* energy from the photons.

$$P_{IC} = -\frac{dE_e}{dt} = \sigma_T c (U'_{rad} - U_{rad}) = \sigma_T c \left(\Gamma^2 + \frac{\Gamma^2\beta^2}{3} - 1\right) U_{rad} = \frac{4}{3}\Gamma^2\beta^2\sigma_T c \boxed{U_{rad}}$$

Incoming power

$$\Gamma^2 - 1 = \Gamma^2\beta^2$$

Independent of spectra of incident photons, only radiation density matters

Similar to $\langle P_{sync} \rangle = \frac{4}{3} c\sigma_T \left(\frac{m_e}{m}\right)^2 \Gamma^2 u_B$

Inverse Compton scattering

Energy losses - generalization

General correction including the energy exchange in the scattering (Blumenthal and Gould 1970)

$$P_{IC} = \frac{4}{3}\Gamma^2\beta^2\sigma_T c U_{rad}\left[1 - \frac{63}{10}\frac{\Gamma}{m_e c^2} + \frac{\langle\epsilon_i^2\rangle}{\langle\epsilon_i\rangle}\ldots\right]$$

Spectral shape matters here

- If electrons are distributed as a power-law $N_e(\Gamma) = C\gamma^{-p}$

$$P_{IC} = \frac{4}{3}\Gamma^2\beta^2\sigma_T c U_{rad}\frac{C}{3-p}(\Gamma_{max}^{3-p} - \Gamma_{min}^{3-p})$$

$$P_{IC}^{KN} \propto \sigma_T \Gamma^2 U_{rad} F(\Gamma)$$

- If electrons have thermal distribution $\langle\beta^2\rangle = \dfrac{3k_BT}{m_c c^2}$

$$P_{IC} = \left(\frac{4k_BT}{m_e c^2}\right)\sigma_T c N_e U_{rad}$$

Inverse Compton scattering

Cooling time

$$P_{IC} = \frac{4}{3}\Gamma^2\beta^2\sigma_T c U_{rad}$$ **— Thomson limit**

$$t_{loss} = \frac{E_e}{|dE_e/dt|} = \frac{\Gamma m_e c^2}{P_{IC}} \simeq 3 \times 10^7 \frac{1}{\Gamma U_{rad}} \mathrm{s}$$

$$P_{IC} = \frac{4}{3}\Gamma^2\beta^2\sigma_T c U_{rad}\frac{C}{3-p}(\Gamma_{max}^{3-p} - \Gamma_{min}^{3-p}) \propto \Gamma^2 U_{rad} F(\Gamma)$$ **— Approximation for PL**

$$t_{loss} = \frac{E_e}{|dE_e/dt|} = \frac{\Gamma m_e c^2}{P_{IC}} \simeq 3 \times 10^7 \frac{1}{\Gamma U_{rad} F(\Gamma)} \mathrm{s}$$

The loss time increase as a function of Γ

Inverse Compton scattering vs Synchrotron

Electrons cooling time

Synchrotron:

$$t_{cool}(\Gamma) = 7.8 \times 10^8 \frac{1}{B^2\Gamma}\ \mathrm{s}$$

Compare with:

Inverse Compton:

$$t_{loss} \simeq 3 \times 10^7 \frac{1}{\Gamma U_{rad} F(\Gamma)} \mathrm{s}$$

$F(\Gamma)$ e.g. from Khangulyan+2014

$$\frac{t_{IC}}{t_{sync}} \propto \frac{U_B}{U_{rad}}$$

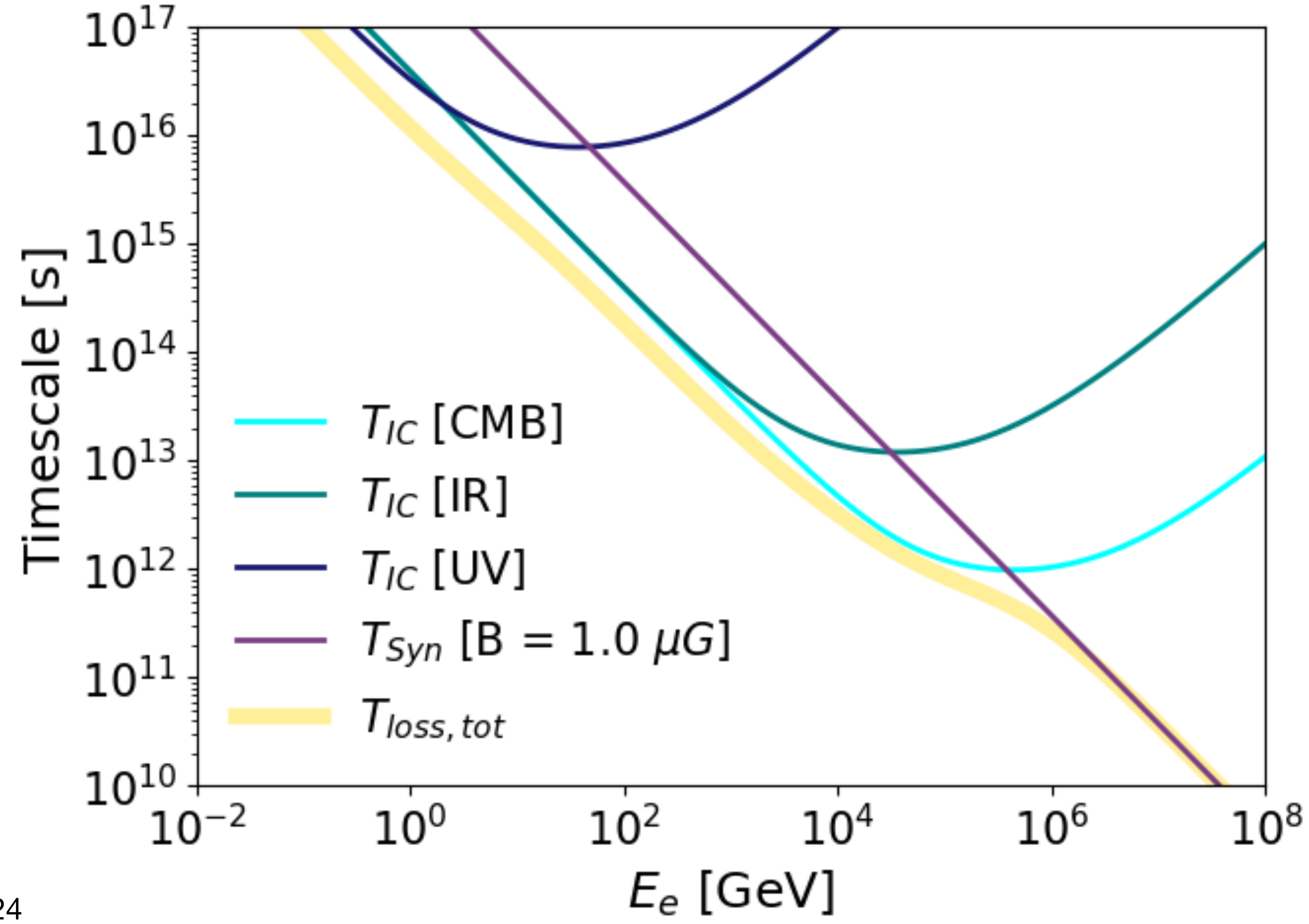
Average fields in the ISM

CMB: $U_{rad} = 0.25\ \mathrm{eV\ cm^{-3}}$; $T = 2.7$ K

IR: $U_{rad} = 0.25\ \mathrm{eV\ cm^{-3}}$; $T = 30$ K

UV: $U_{rad} = 0.37\ \mathrm{eV\ cm^{-3}}$; $T = 3 \times 10^4$ K

$B = 1\mu G$



Inverse Compton scattering

Emission spectrum

In each scattering the photon is up-scattered from an energy ϵ_i to an energy ϵ_f

The **energy gain depends on** the relative angle of collision θ_i and on the scattering angle α'

The probability of ending up in α' is given by the cross section $\dfrac{d\sigma}{d\Omega}$

$$\epsilon_f = \frac{\epsilon_f'}{\Gamma(1-\beta\cos\theta_f)} = \Gamma\epsilon_f' \quad *$$

$$\cos\theta_f \sim \beta$$

$$1-\beta^2 = \Gamma^{-2}$$

Thomson condition:

$$* \; \Gamma\epsilon_f' = \Gamma\epsilon_i' = \Gamma^2\epsilon_i(1-\beta\cos\theta_i)$$

in lab.frame scattered photon follows electron propagation direction, i.e., $\theta_i \to \alpha$

$$\dot{N}_f(\epsilon_f,\Omega_f) = \frac{dN_{ph}}{d\Omega_f dt d\epsilon_f} = c\int d\epsilon_i \int d\Omega_{ph,i} \int d\Gamma \int d\Omega_e (1-\beta\cos\alpha)\frac{N_{e,0}}{4\pi}\Gamma^{-p} \times$$

$$* \cos\alpha \sim \cos\theta_i$$

$$\times \frac{N_{ph,0}\delta(\epsilon_i-\epsilon_0)}{4\pi}\sigma_T \delta(\epsilon_f - \epsilon_i\Gamma^2(1-\beta\overset{*}{\cos\alpha}))\delta(\Omega_{ph,f}-\Omega_e)$$

Assumpion: final direction of photon = initial direction of electron

Inverse Compton scattering

Emission spectrum

In each scattering the photon is up-scattered from an energy ϵ_i to an energy ϵ_f

The **energy gain depends on** the relative angle of collision θ_i and on the scattering angle α'

The probability of ending up in α' is given by the cross section $\boxed{\frac{d\sigma}{d\Omega}}$

Consider all possible angles

Distribution of electrons

$$\dot{N}_f(\epsilon_f, \Omega_f) = \frac{dN_{ph}}{d\Omega_f dt d\epsilon_f} = c \int d\epsilon_i \int d\Omega_{ph,i} \int d\Gamma \int d\Omega_e (1 - \beta \cos\alpha) \frac{N_{e,0}}{4\pi} \Gamma^{-p} \times$$

Monochromatic photon field:

Energy exchange

$^*\cos\alpha \sim \cos\theta_i$

$$\times \frac{N_{ph,0}\delta(\epsilon_i - \epsilon_0)}{4\pi} \boxed{\frac{d\sigma}{d\Omega}} \delta(\epsilon_f - \epsilon_i \Gamma^2 (1 - \beta \overset{*}{\cos}\alpha)) \delta(\Omega_{ph,f} - \Omega_e)$$

Assumpion: final direction of photon = initial direction of electron

Compton scattering
Cross section

Klein-Nishina treatment of inelastic scattering

Klein-Nishina cross section (QED)

$$\frac{d\sigma}{d\Omega} = \frac{3}{16\pi}\sigma_T \left(\frac{\epsilon_f}{\epsilon_i}\right)^2 \left(\frac{\epsilon_i}{\epsilon_f} + \frac{\epsilon_f}{\epsilon_i} - \sin^2\theta\right)$$

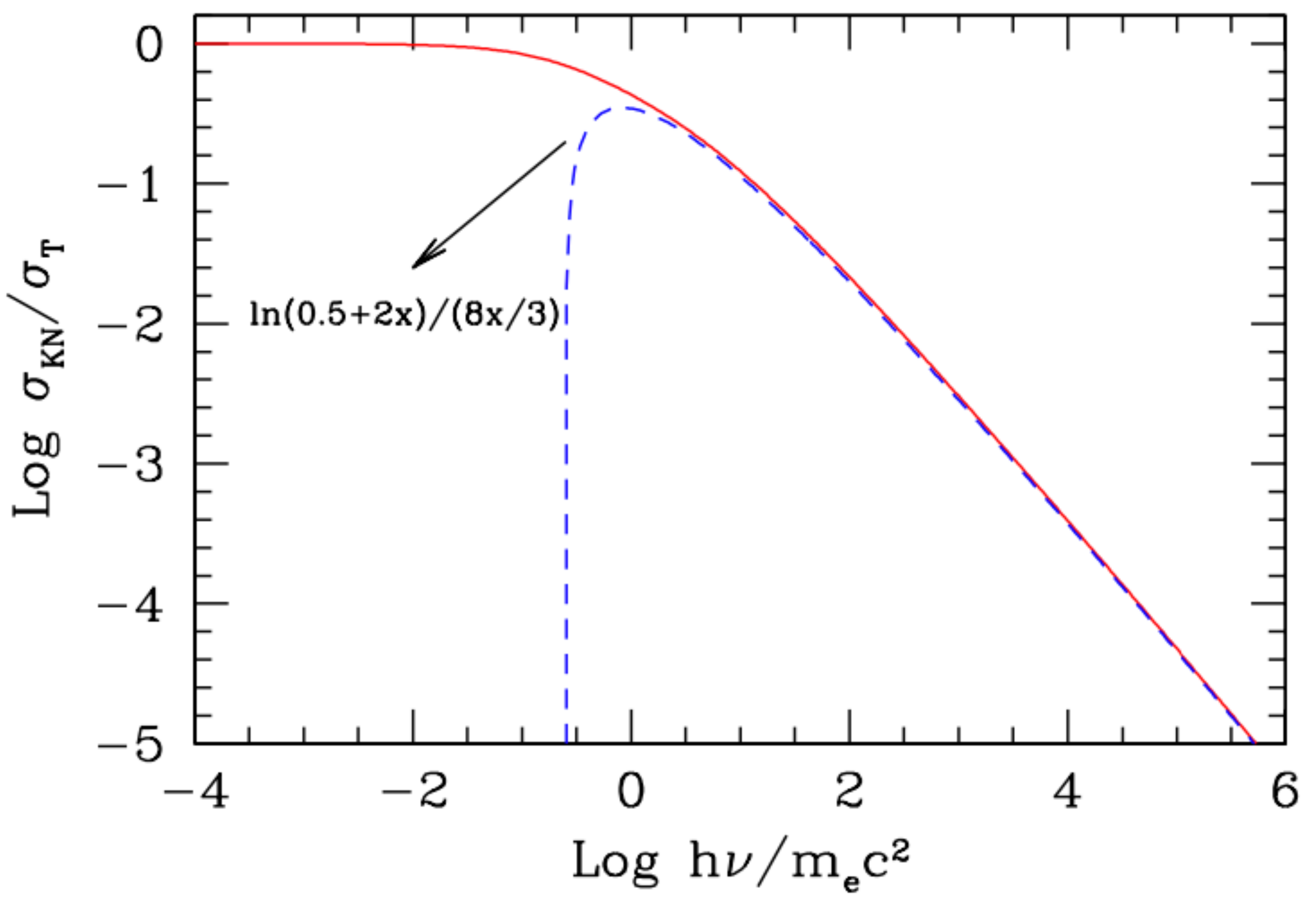
- Forward scattering is favored at higher energies

Integrating…

$$x = \frac{h\nu}{m_e c^2}$$

$$\sigma = 2\pi \int_0^\pi \frac{d\sigma}{d\Omega} \sin\theta d\theta = \frac{3}{4}\sigma_T \left[\frac{1+x}{x^3}\left(\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right) + \frac{1}{2x}\ln(1+2x) - \frac{1+3x}{(1+2x)^2}\right]$$

- At high energies the cross section is reduced compared to classical Thomson cross section



$$\begin{cases} \sigma(x) \simeq \sigma_T(1 - 2x + \dots) & x = \dfrac{h\nu}{m_e c^2} \ll 1 \\ \sigma(x) \simeq \dfrac{3}{8}\sigma_T \dfrac{1}{x}\left(\ln 2x + \dfrac{1}{2}\right) & x = \dfrac{h\nu}{m_e c^2} \gg 1 \end{cases}$$

Inverse Compton scattering

Emission spectrum: relativistic electrons over mono-energetic photons

$$\dot{N}_f(\epsilon_f, \Omega_f) = \frac{dN_{ph}}{d\Omega_f dt d\epsilon_f} = c \int d\epsilon_i \int d\Omega_{ph,i} \int d\Gamma \int d\Omega_e (1 - \beta \cos\alpha) \frac{N_{e,0}}{4\pi} \Gamma^{-p} \times$$

$$\times \frac{N_{ph,0}\delta(\epsilon_i - \epsilon_0)}{4\pi} \sigma_T \delta(\epsilon_f - \epsilon_i \Gamma^2 (1 - \beta\cos\alpha)) \delta(\Omega_{ph,f} - \Omega_e)$$

$$\beta = 1$$

$$x = \epsilon_0 \Gamma^2 (1 - \cos\alpha)$$

$$\frac{dx}{d\Gamma} = \frac{2x}{\Gamma}$$

$$\propto \int \Gamma^{-p} \delta(\epsilon_f - \epsilon_0 \Gamma^2 (1 - \cos\alpha)) d\Gamma$$

$$\frac{-p+1-2}{2} = \frac{-(p+1)}{2}$$

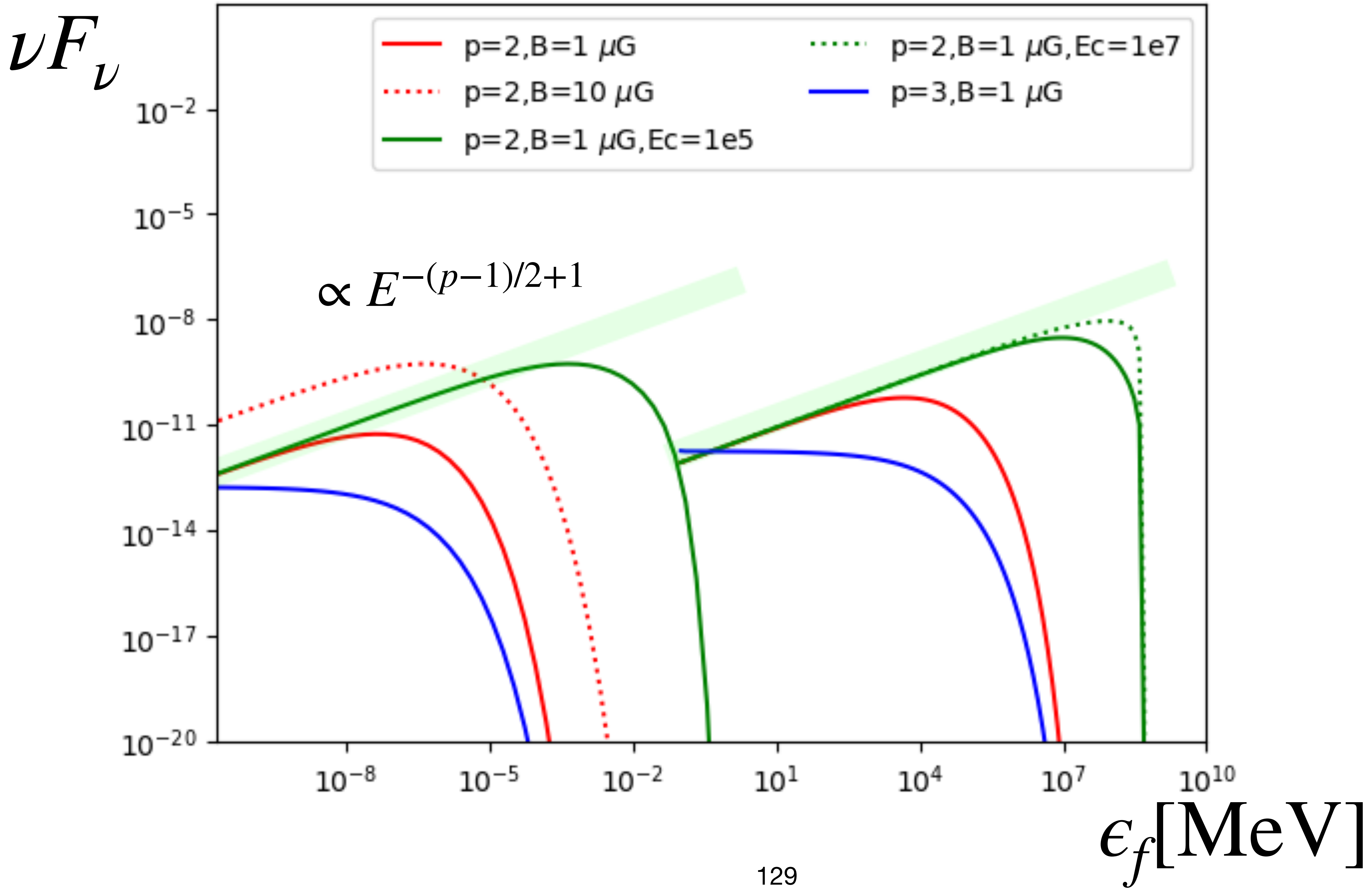
$$\propto \int \Gamma^{-p+1} \delta(\epsilon_f - x) \frac{dx}{2x} \propto \frac{1}{2} \int x^{\frac{-p+1}{2} - 1} \delta(\epsilon_f - x) dx \propto \epsilon_f^{\frac{-(p+1)}{2}}$$

$$\dot{N}_f \propto \epsilon_f^{\frac{-(p+1)}{2}} \implies \epsilon_f \dot{N}_f \sim F_{IC} \propto \epsilon_f^{\frac{-(p+1)}{2}+1} = \boxed{\epsilon_f^{\frac{-(p-1)}{2}}}$$

$$\left[F_{\nu,sync} \propto \nu^{\frac{-(p-1)}{2}}\right]$$

Inverse Compton vs. Synchrotron

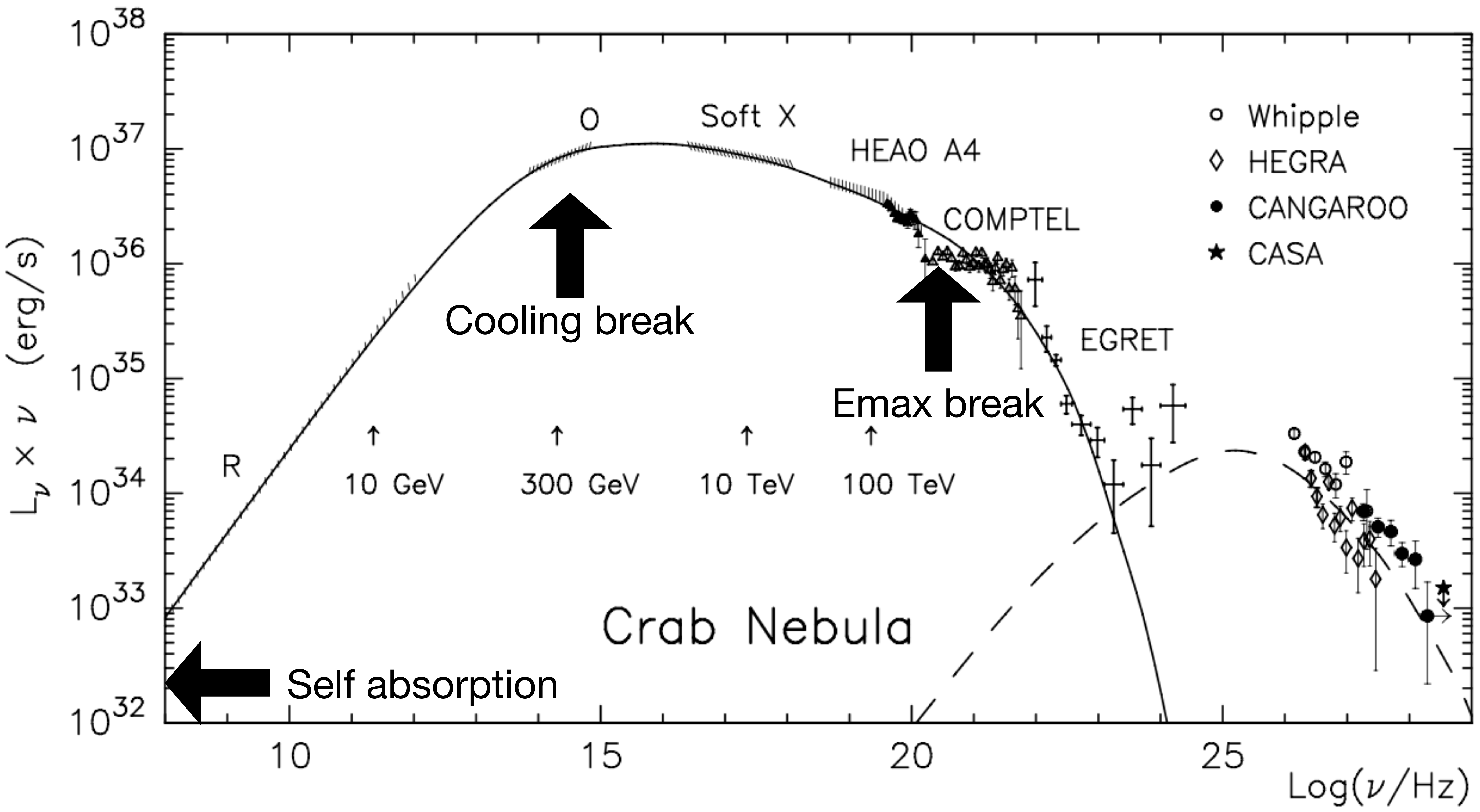
$F_{IC} \propto \epsilon_f^{-(p-1)/2}$ // $F_{sync} \propto \nu^{-(p-1)/2}$



Inverse Compton vs. Synchrotron

$F_{IC} \propto \epsilon_f^{-(p-1)/2}$ // $F_{sync} \propto \nu^{-(p-1)/2}$ *

* In the energy range.. not affected by self absorption, cooling, Klein Nishina, where monochromatic photon fields counts



Inverse Compton + Synchrotron

A special photon field: Synchrotron Self Compton (SSC)

Synchrotron emission produces a power-law spectrum $\propto \nu^{-\alpha}$. If radiation interacts with the relativistic electrons we have Synchrotron self Compton effect. Electrons see the photons with ν'

Synchrotron photon field

All electrons

$$P_{IC} \propto \nu \int d\nu' \frac{\nu'^{-\alpha}}{\nu'} \int d\Gamma \Gamma^{-p} \delta(\nu - \frac{4}{3}\Gamma^2 \nu')$$

Average frequency after IC scattering

$\alpha = \frac{(p+1)}{2}$

$$\propto \nu \int \int d\nu' d\Gamma \nu'^{-\alpha-1} \Gamma^{-2\alpha-1} \delta(\nu - \frac{4}{3}\Gamma^2 \nu')$$

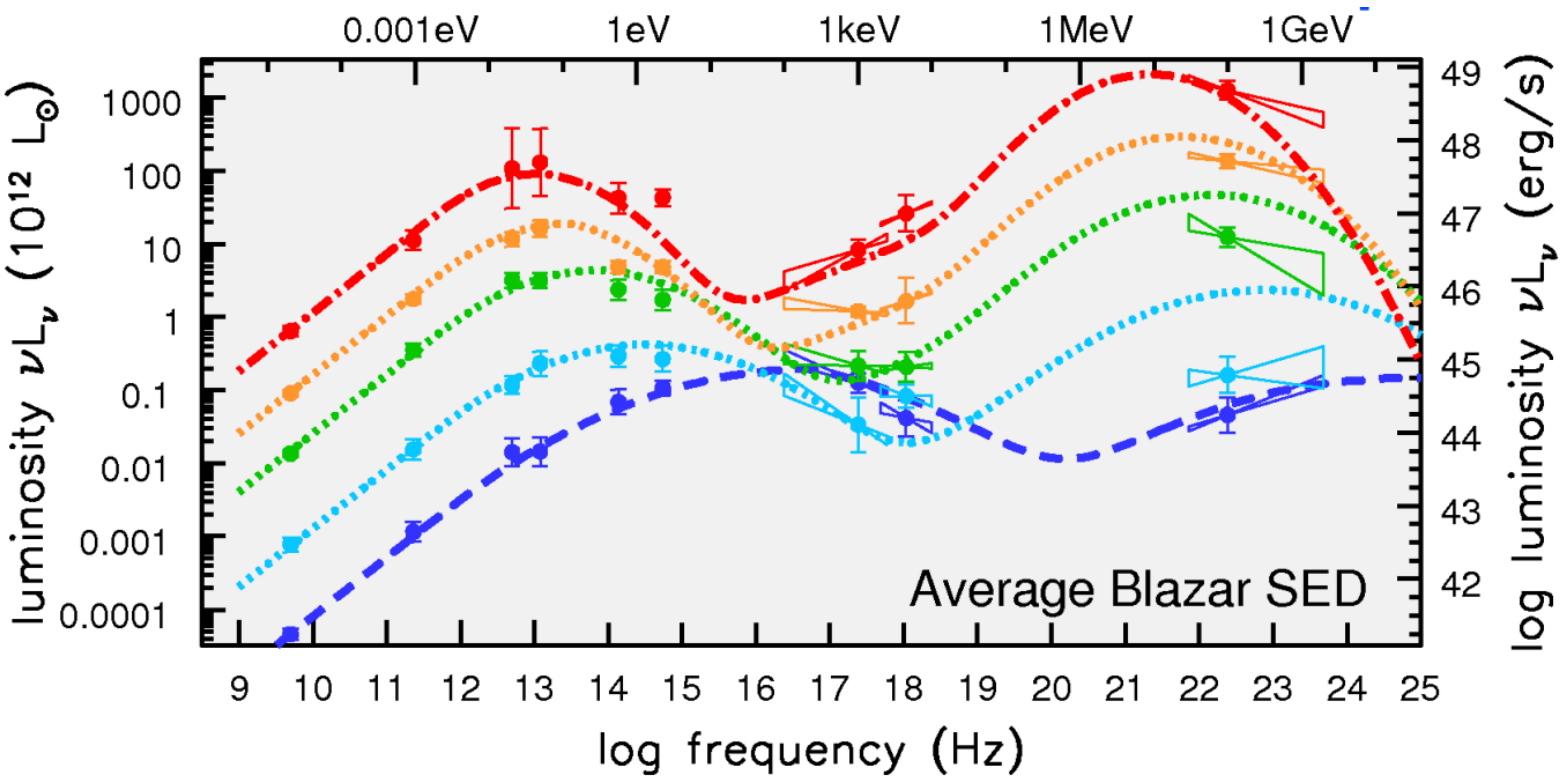
$x = \frac{4}{3}\Gamma^2 \nu'$

$$\propto \nu \int \int dx d\Gamma \nu'^{-\alpha-1} \Gamma^{-2\alpha-1} \delta(\nu - x) \propto \nu \int dx x^{-\alpha-1} \delta(x - \nu) \propto \nu^{-\alpha}$$

SSC have the same slope as synchrotron

Inverse Compton + Synchrotron

A special photon field: Synchrotron Self Compton (SSC)



Take home messages

- Hadronic and Leptonic emission characterize the high-energy spectra
- **Hadronic processes** have a characteristic energy **threshold** and their spectrum reflects the spectrum of particles; hadrons do **not** suffer **significant energy losses** at high energies.
- **Leptonic processes** are present throughout the **entire electromagnetic spectrum.** Leptonic interactions shape the spectrum of parent electrons. **Losses are energy dependent**.
  - **Synchrotron** emission is important **from radio to X-rays,** tells us information on the parent electron spectrum, but we need to account for self-absorption and spectral distortion due to cooling;
  - **Inverse Compton** in the Thomson limit mimics the shape of the Synchrotron emission; in the Klein Nishina limit the spectrum falls off quickly, the process is less efficient but more catastrophic. **KN** is important for **TeV electrons** in the presence of enhanced **IR radiation**;
  - **Bremsstrahlung** can emerge only in very **dense environments where Ne>Np,** or below threshold for pion production and is in general subdominant at TeV energies.

Practical summary

	Energy of the photons	Gamma-ray slope	Tloss
pp-interaction	10 % parent particles	~ p	$5.4 \times 10^7 \left(\frac{n}{\mathrm{cm}^{-3}}\right)^{-1} \mathrm{yr}$
Bremsstrahlung	~50% parent particles	~p	$4 \times 10^7 \left(\frac{n}{\mathrm{cm}^{-3}}\right)^{-1} \mathrm{yr}$
Synchrotron	$\nu_c \simeq 280\ \Gamma^2 \left(\frac{B}{10^{-4}\ \mathrm{G}}\right) Hz$	-(p-1)/2 (+1) (SED)	$7.8 \times 10^8 \frac{1}{B^2 \Gamma}\ \mathrm{s}$
Inverse Compton	$\epsilon_f \propto \Gamma^2 \epsilon_i$ $\epsilon_f \propto \Gamma m_e c^2$	-(p-1)/2 (+1) (SED) [Before KN]	$3 \times 10^7 \frac{1}{\Gamma U_{rad} F(\Gamma)} \mathrm{s}$