Simulating light in scattering dominated media

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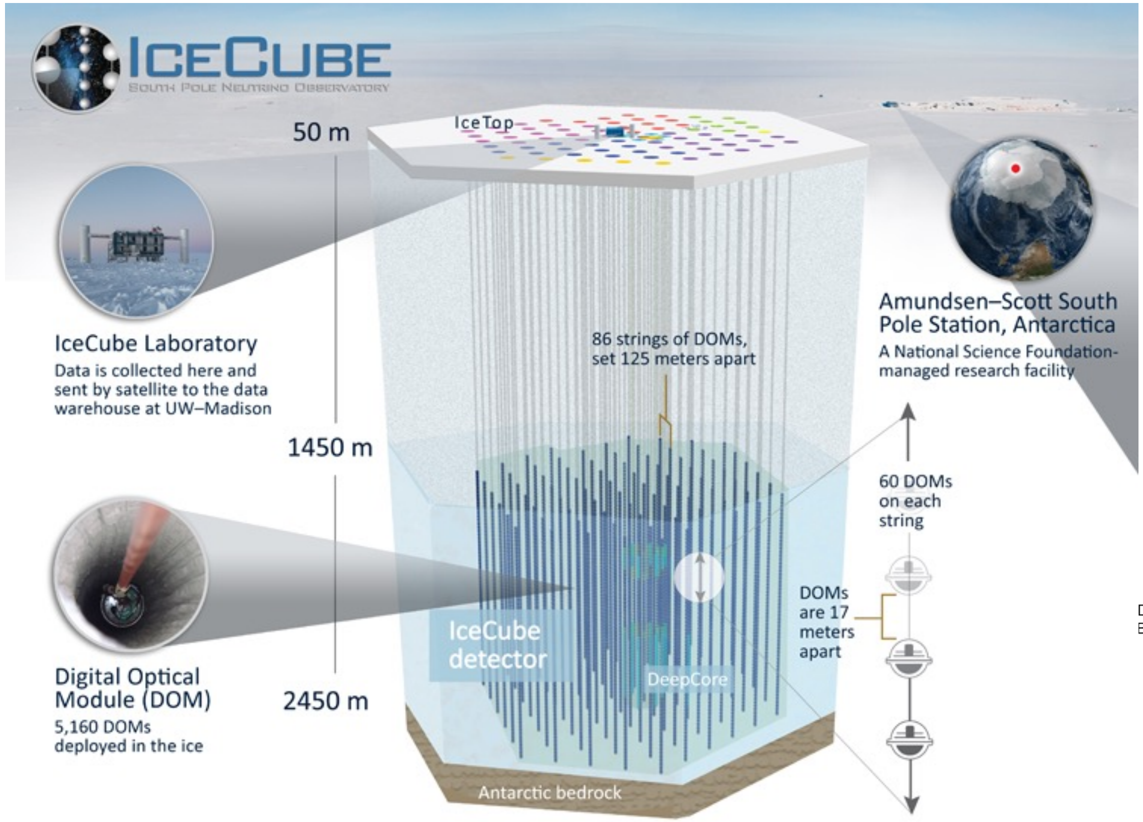
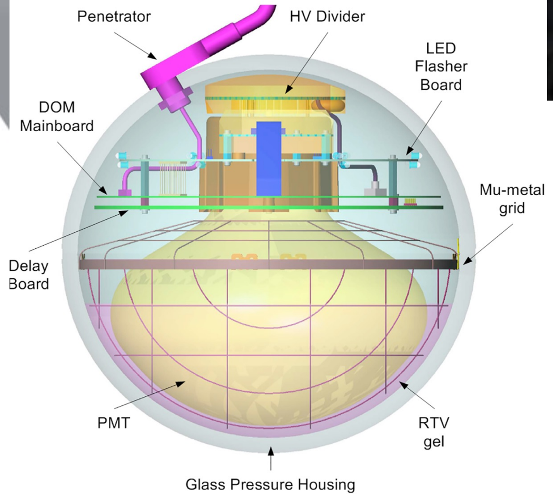
IceCube

Gigaton neutrino detector located at the south pole





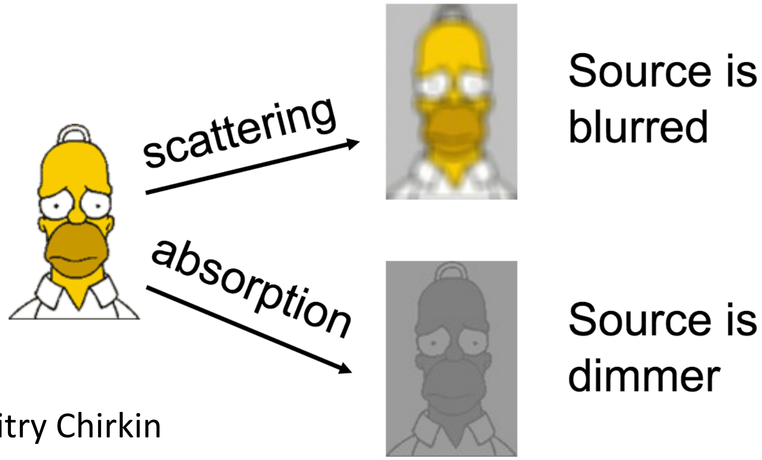
IceCube

Muon neutrinos interact with the surrounding ice/rock and produce muons that travel through the detector.

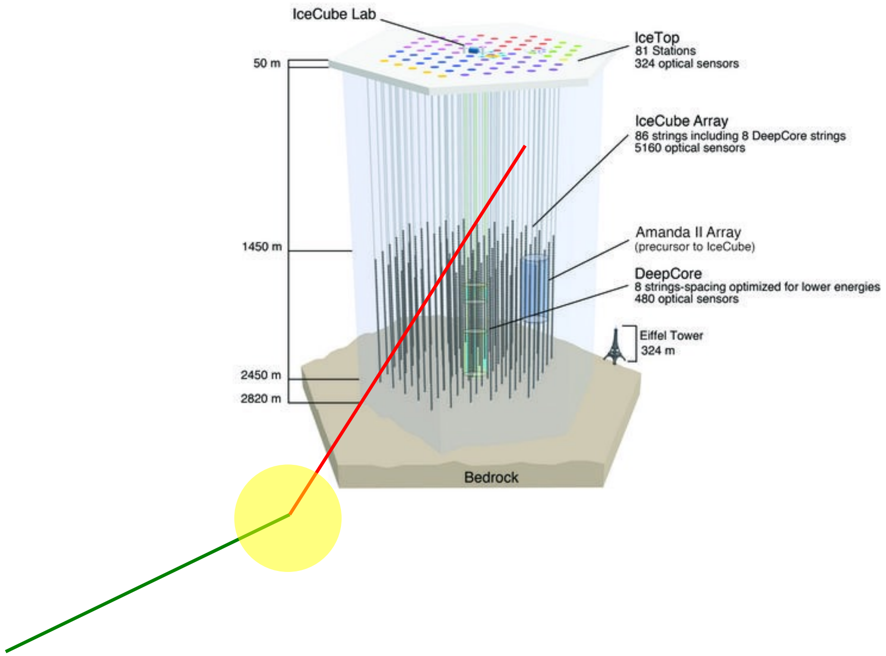
- Produces Cherenkov light as it travels.

Cherenkov light is scattered and absorbed

- Effects the angular and energy resolution.



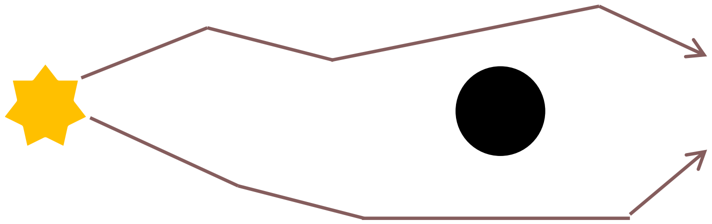
Example by Dmitry Chirkin



Motivation

IceCube uses path tracing to propagate light in the ice.

- However, most rays never reach a DOM.



Path tracers can be run *backwards* in time, but then most rays will never reach a light source.

- Path tracers can't constrain both the starting and ending location of the rays.

The fundamental problem is that the interesting paths are highly constrained.

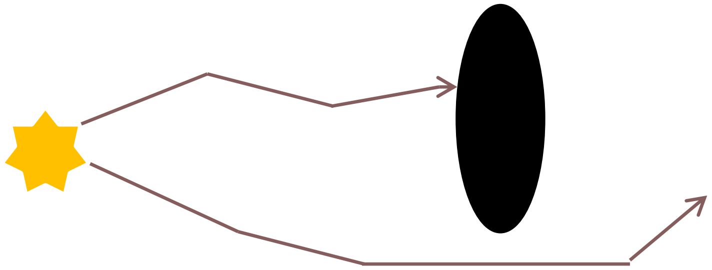
- Is there another way to approach this?

Approximations

DETECTOR BIAS

Make the detectors much larger to capture more rays.

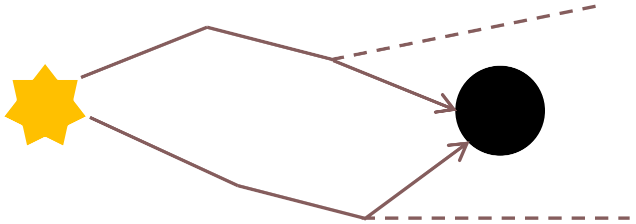
- Introduces bias that can't be corrected for.



PATH BIAS

Adjust the path the light takes to bias it towards the detector.

- Introduces bias that can be corrected for.
- However, can massively increase variance to the point of being useless.



Can we do this without bias?

Path integration

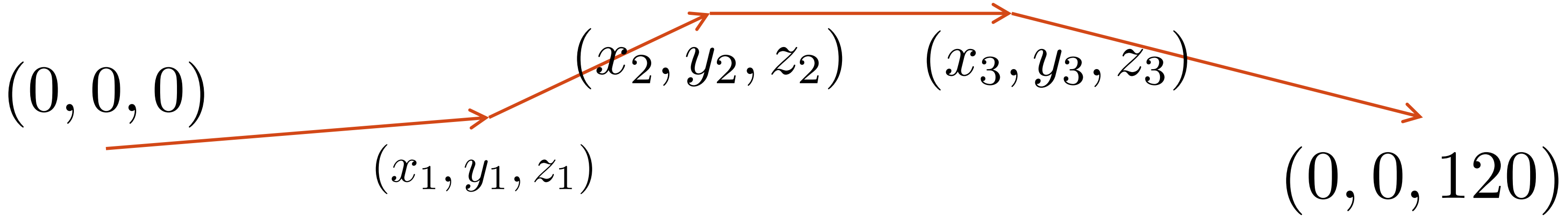
The start and end locations of the ray can be constrained if the problem is specified in terms of a classical path integral.

$$\int_{f \in \Omega} e^{-S[f]} Df$$

Space of all paths

Probability of path 'f'

Eg: $f = \{(0,0,0), (x_1, y_1, z_1), (x_2, y_2, z_2), \ldots, (0,0,120)\}$



Evaluation of the integral

Information can be extracted about the light propagation by framing the integrand as a probability distribution:

$$e^{-S[f]} \to p[f] = p(x_1, y_1, z_1, x_2, y_2, z_2, \ldots)$$

This distribution can be sampled with an MCMC.

More details on the construction of p[f] in the paper:

arXiv.org > hep-ex > arXiv:1811.04156

High Energy Physics – Experiment

Using path integrals for the propagation of light in a scattering dominated medium

Gabriel H. Collin

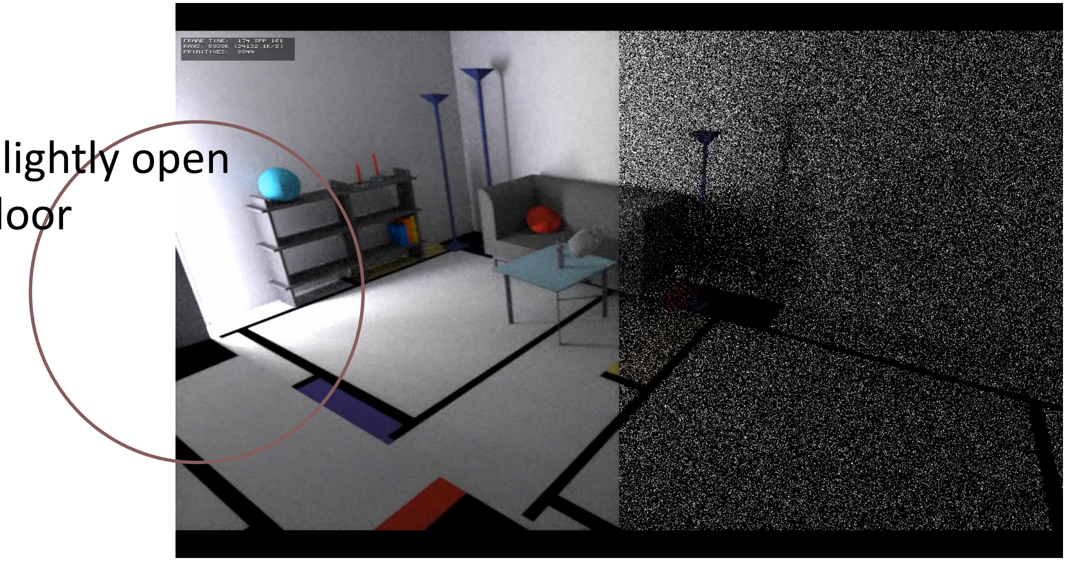
Industry use

This idea inspired by a CGI rendering technique called Metropolis light transport.

- Computer animation often runs into a similar problem to us, where only a small fraction of light paths are detectable.
- Canonical example is a light source in another room that shines through a door that is only slightly cracked open.

CGI industry mainly renders scenes that are dominated by reflections.

- In IceCube, light transport is entirely scattering.



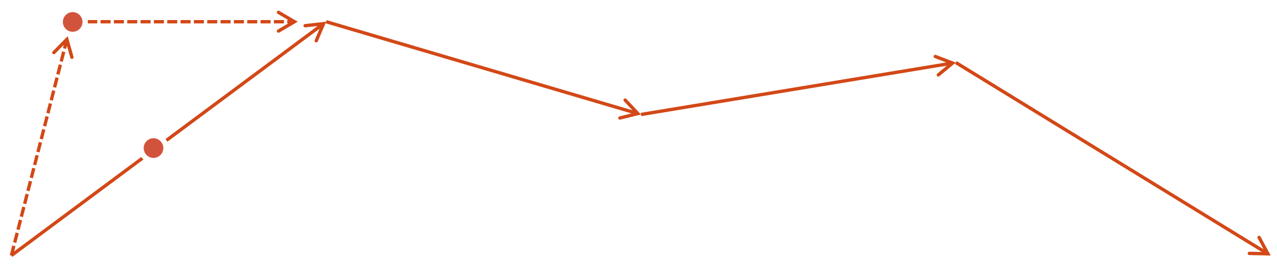
Left: Rendering algorithm similar to Metropolis light transport.
Right: Standard path tracing algorithm.

http://raytracey.blogspot.com.es/2010/12/real-time-metropolis-light-transport-on.html

Reversible jump MCMC

The number of places where light scatters is not fixed.

- Thus, the dimensionality of the probability distribution is variable.



Reversible Jump Markov Chain Monte Carlo can change the number of dimensions in a probability distribution.

Reversible jump MCMC

The acceptance probability is based on the following ratio:

$$\frac{p_1(\vec{\phi})}{\boxed{p_0(\vec{\theta})q(\vec{q})}} \frac{P_{1\to 0}}{P_{0\to 1}} \left| \frac{\partial g(\vec{\theta}, \vec{q})}{\partial(\vec{\theta}, \vec{q})} \right|$$

$$\vec{\phi} = g(\vec{\theta}, \vec{q})$$

Padded probability distribution

Proposal rates

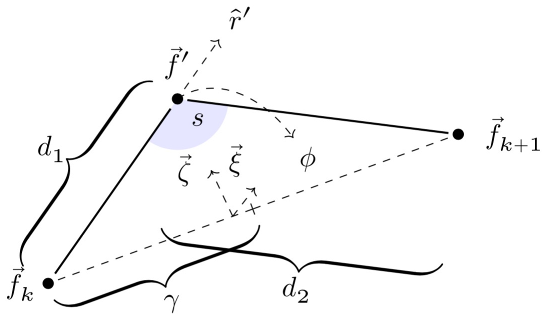
Jacobian of proposal function.

q can be marginalised out later for free.

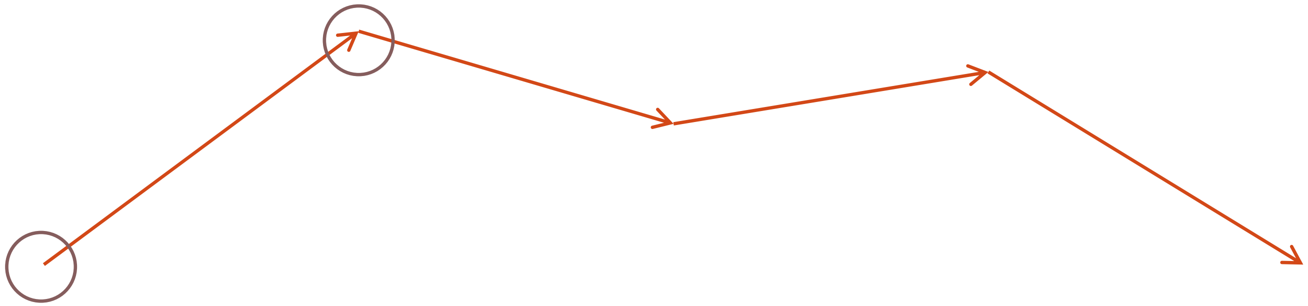
Reversible jump for light propagation

A path with N vertices exists in $\mathbb{R}^{3N}$

- We wish to propose a new path with N+1 vertices.
- Requires a *q* with 3 parameters, and a choice of *g*.



g selects a pair of vertices.



Then inserts a new vertex between them.

- Position of new vertex based on three random values from q

Path length distribution

From the samples created by the MCMC, the probability distribution for path length can be easily extracted.

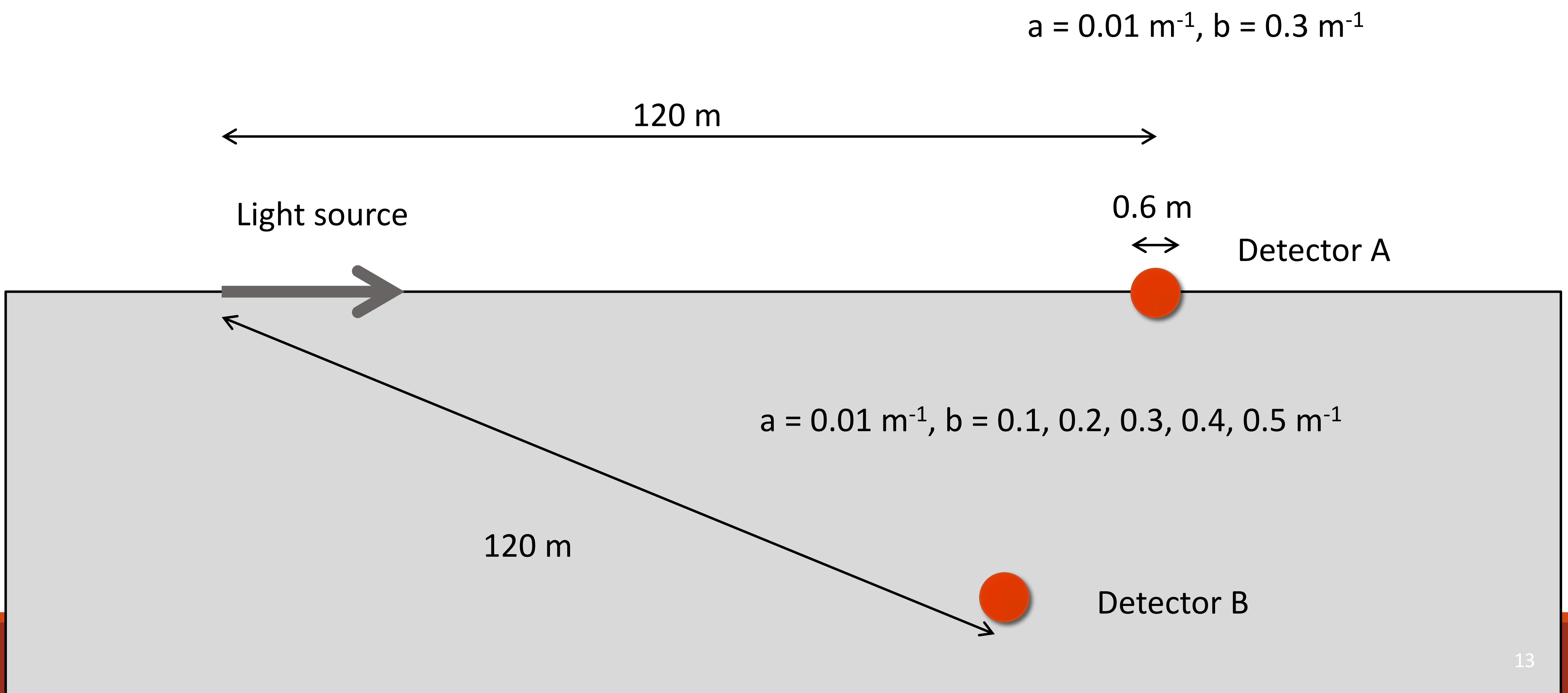
- IceCube measures photon arrival time, which is directly related to path length.
- $P(L < X)$ = fraction of samples where the length of the path is less than X.

To validate the method, the length distribution produced by the path sampler can be compared to one created using a ray tracer.

An MCMC usually requires a burn-in period, however this can be partially avoided by seeding the MCMC with the ray-tracer.

Synthetic test case

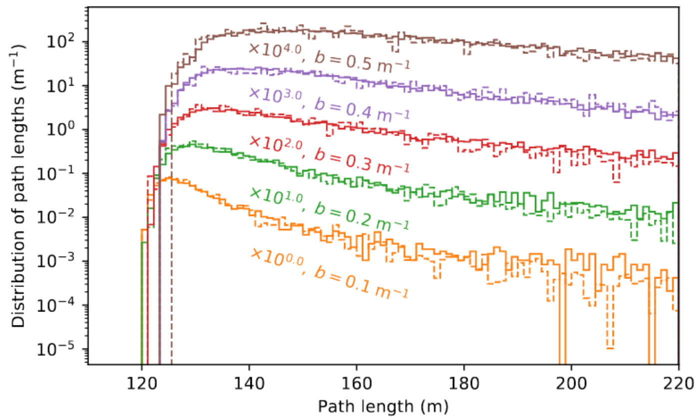
One light source, with two detectors



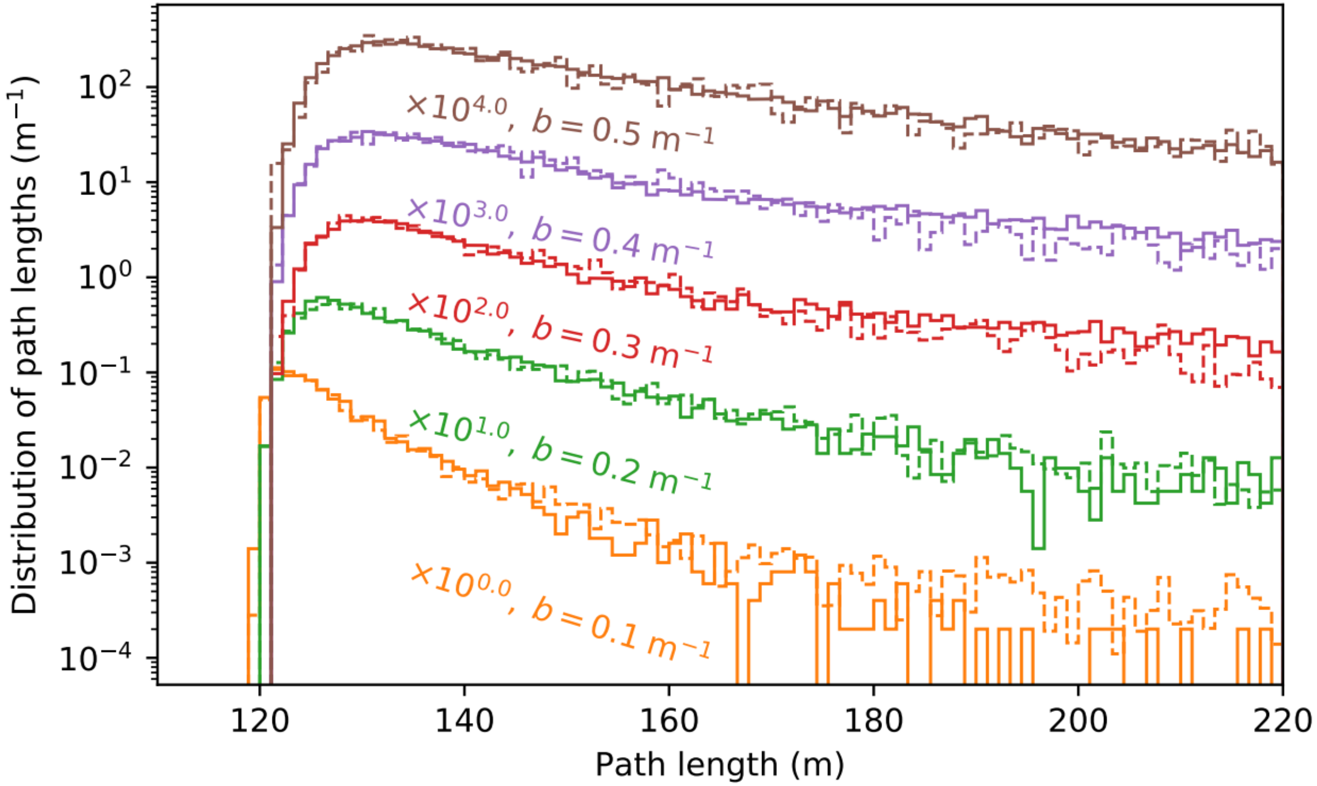
Path length distribution

Solid: path sampler. Dashed: reference ray tracer

- Ray tracer was run until 5000 samples collected.
- Path sampler was run until results matched the path sampler.



Detector B



Detector A, Acceptance rate ~20%

Lines offset for readability

Performance

Ray tracer is also CPU based to allow a performance comparison.

b	Ray tracer	Path sampler
0.1 m^{-1}	~46000 s	~23 s
0.2 m^{-1}	~78000 s	~74 s
0.3 m^{-1}	~99000 s	~232 s
0.4 m^{-1}	~122000 s	~373 s
0.5 m^{-1}	~156000 s	~416 s

Performance improvement of 300 to 1000 times faster.

- The b = 0.3 to 0.5 m^{-1} cases are probably most comparable to conditions in IceCube.

Other applications

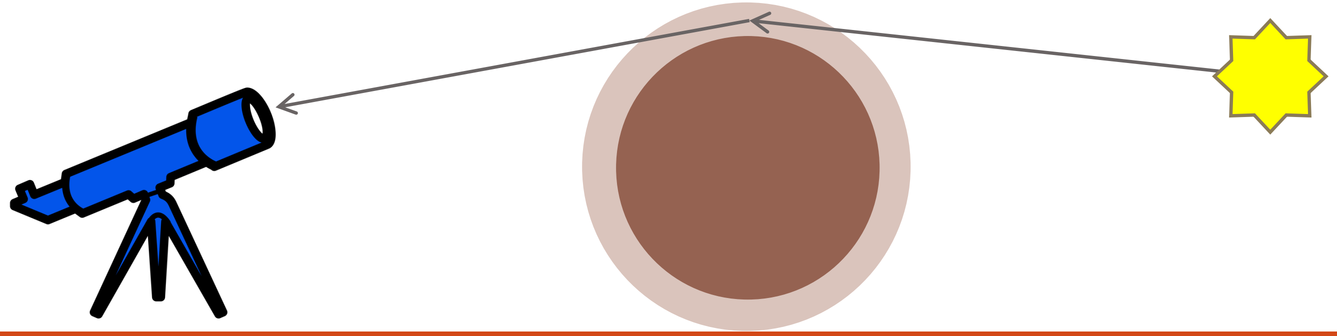
This approach to simulation is useful when initial and final states are highly constrained.

Litmus test:

- Are you throwing out the vast majority of your events (99.9%+) due to them not meeting one of these constraints?

Constraints do not have to just be in position.

- Eg: initial and final angle for light passing through a planetary atmosphere.



Other applications

Path does not just have to describe light.

- Eg: Simulation of transport of neutrons.

Constraints could be discrete parameters.

- Eg: Simulation of atmospheric showers.
  - Initial condition: particle must be a nucleus.
  - Final condition: shower products must reach underground detector.

May also be possible to incorporate selection cuts into the constraint.

Disadvantages

Relative and especially absolute light yields are difficult to calculate.

Relative light yield is given by the ratio of normalisations for each detector.

- This is equivalent to finding a Bayes factor in Bayesian inference.

We can use the geometric estimator:

$$\mathcal{B} = \frac{\mathbb{E}_A[\sqrt{p_B(x)/p_A(x)}]}{\mathbb{E}_B[\sqrt{p_A(x)/p_B(x)}]}$$

To estimate the variance on the light yield

- Ray tracer and path sampler were run four times. (standard deviation in parentheses.)

b	Ray tracer	Path sampler
0.1 m^{-1}	0.67(3)	0.64(7)
0.2 m^{-1}	0.79(2)	0.77(1)
0.3 m^{-1}	0.73(2)	0.76(7)
0.4 m^{-1}	0.68(2)	0.64(4)
0.5 m^{-1}	0.59(2)	0.50(6)

- Path sampler has higher variance.

A new approach?

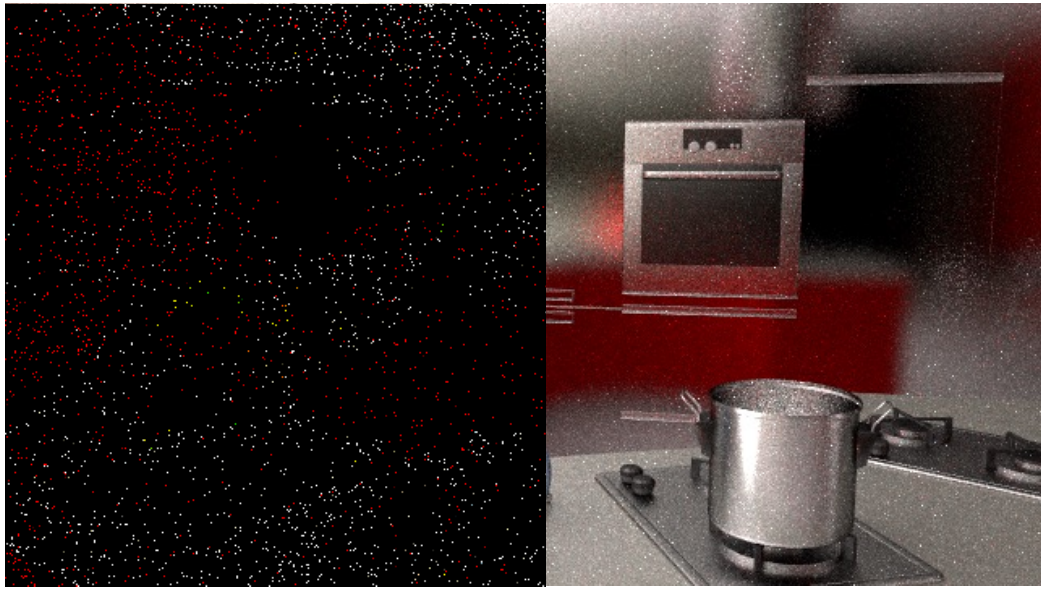
Can we combine the advantages of path tracing and metropolis light transport?

- Perhaps with Guided Path Tracing.

Run path tracing in batches.

- Use the "useful" rays in each batch to bias the next batch.
- Results in minimal biasing, and therefore minimal variance.
- Bias can be corrected for like event-weighting.

A further advantage is that this can slot into many existing physics simulations.



Left: Path tracing. Right: Guided path tracing.

Conclusion

Simulation of light can be posed as a path integral from which samples can be drawn.

Reproduces the timing distribution of light incident on a detector.

- Up to 1000 times faster than a ray tracer in synthetic test case.

Method is generally applicable to a wide range of problems.

- When initial and final states are highly constrained.

Backup

Angular scattering distribution

Distribution

$$\sigma(\cos\psi) = f_{\mathrm{SL}} p_{\mathrm{SL}}(\cos\psi) + (1 - f_{\mathrm{SL}}) p_{\mathrm{HG}}(\cos\psi),$$

Simplified Liu:

$$p_{\mathrm{SL}}(\cos\theta) = \frac{1}{2}\frac{1+g}{1-g}\left[\frac{1+\cos\theta}{2}\right]^{\frac{2g}{1-g}}$$

Henyey-Greenstein:

$$p_{\mathrm{HG}}(\cos\theta) = \frac{1}{2}\frac{1-g^2}{(1+g^2-2g\cos\theta)^{3/2}}$$

Detection probability

Conditional detection probability:

$$\rho(\vec{f}_{n-1}) = \exp\left(3\cos\omega - \ln\cosh(2\cos\omega + 0.7) - 1\right)$$

$$\cos\omega = \hat{\rho} \cdot \frac{\vec{f}_{n-1} - \vec{\eta}}{|\vec{f}_{n-1} - \vec{\eta}|}$$

Chosen to follow IceCube DOM angular response.

Jump distributions

$$q(s) = \frac{\beta e^{-\beta \cos s}}{2 \sinh \beta} \sin s,$$

$$q(t) = (2 + 2 \cosh t)^{-1},$$

$$q(\phi) = \frac{1}{2\pi},$$

Jump rates

$$p(n \to n+1, k) = \frac{\tau_b(k)}{\sum_{l=1}^{n-1} \tau_b(l)},$$

$$p(n \to n-1, k) = \frac{1}{n-2}.$$

Incident angle distribution

For a smoothly varying b: