Unbinned test for anomalous dispersion

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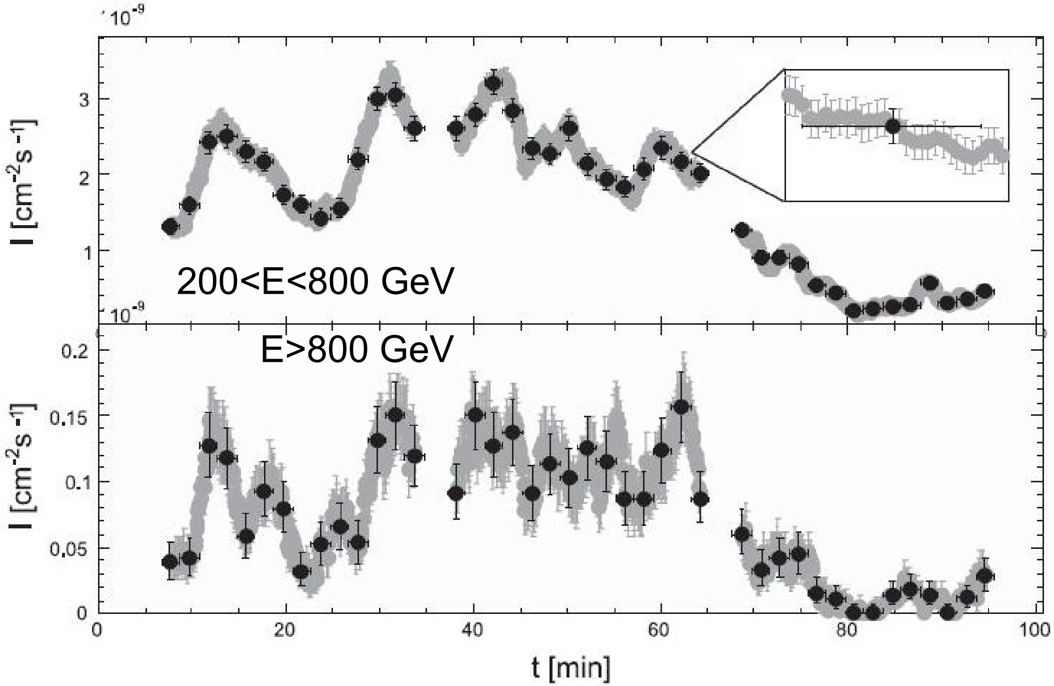
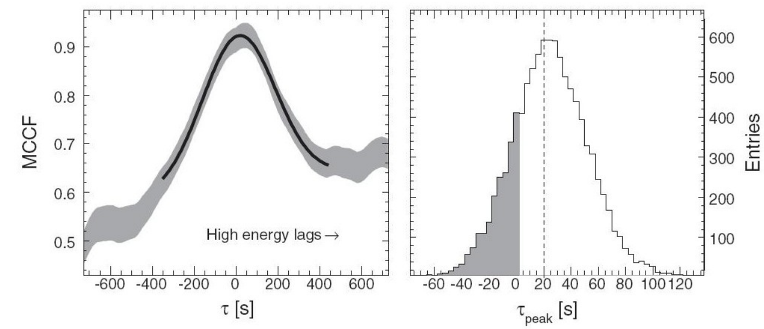
- Motivation
- Description of the method
- Performance of the method
- Prospects for CTA

Durham University

Why an unbinned test?



H.E.S.S. Collaboration *PRL* **101**, 170402 (2008).



Bin on minute timescales.
Move bins in 5s steps, cross correlate.

$$\delta t \approx \xi \frac{\Delta E}{E_{Pl}} \frac{L}{c}$$

For PKS 2155-304 at z~0.113,
δt~4.26 s per TeV
Δt$\geq$300s

since delay << flare/burst width want to probe maximum time resolution of events with unbinned methods, define sensitivity of test as

$$\eta = \frac{\delta t}{\Delta t}$$

Why a non-parametric test?

Much cleverer people than I can describe the intrinsic light curve shape & how much it gets changed by propagation through the intervening cosmos...

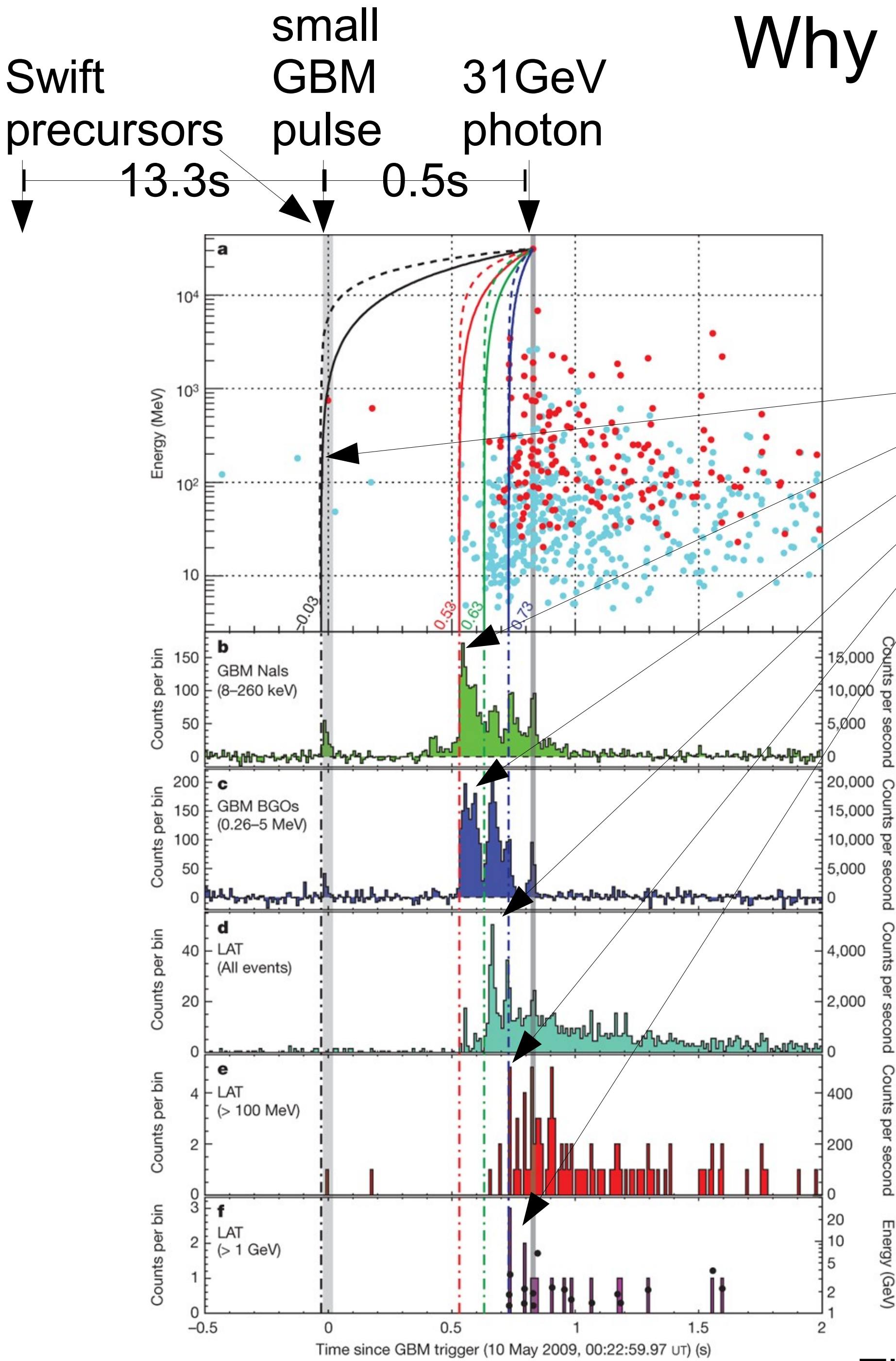
$$\frac{dP}{dEdt} = N \int_0^\infty \mathit{\Gamma}(E_s)\, C(E_s, t)\, G(E - E_s, \sigma_E(E_s))\, F_s(t - D(E_s, M_{QG}, Z))\, dE_s$$

- $\frac{dP}{dEdt}$: probability density function
- $\mathit{\Gamma}(E_s)$: energy distribution at source: power law? broken power law? curvature?
- $C(E_s, t)$: effective area
- $G(E - E_s, \sigma_E(E_s))$: energy resolution of instrument
- F_s: time distribution at source
- $D(E_s, M_{QG}, Z)$: propagation delay

Martinez & Errando *APh* **31**, 226 (2009).

but there are just so many uncertainties to take into account.

Why an unbinned, non-parametric test?



Fermi Collaboration *Nature* **462**, 331 (2009).

Table 2 | Limits on Lorentz Invariance Violation

#	$t_{start} - T_0$ (ms)	Limit on $\lvert\Delta t\rvert$ (ms)	Reasoning for choice of t_{start} or limit on Δt or $\lvert\Delta t/\Delta E\rvert$	Lower limit on $M_{QG,1}/M_{Planck}$
(a)◇	−30	< 859	start of any < 1 MeV emission	> 1.19
(b)◇	530	< 299	start of main < 1 MeV emission	> 3.42
(c)◇	*648*	< 181	start of main > 0.1 GeV emission	> 5.63
(d)◇	730	< 99	start of > 1 GeV emission	> 10.0
(e)♦	—	< 10	association with < 1 MeV spike	> 102
(f)♦	—	< 19	If 0.75 GeV‡ γ-ray from 1st spike	> 1.33
(g)♣	$\lvert\Delta t/\Delta E\rvert$ < 30 ms/GeV		lag analysis of > 1 GeV spikes	> 1.22

Which part of the emission do you determine the delay from?
All of the delays are constraining, but 10% of short GRBs show pre-cursor activity, Troja, Rosswog & Gehrels *ApJ* **723**, 1711 (2010). If you start from the 1st Swift precursor flare then the limit becomes M_{QG}>0.09M_{Pl}...

Whereas the DisCan method, Scargle, Norris & Bonnell *ApJ* **673**, 972 (2008), is robust against these complications by examining the Shannon information (reverse entropy) of all the photons in the flare.

The energy resolution of an IACT is prohibitive though...

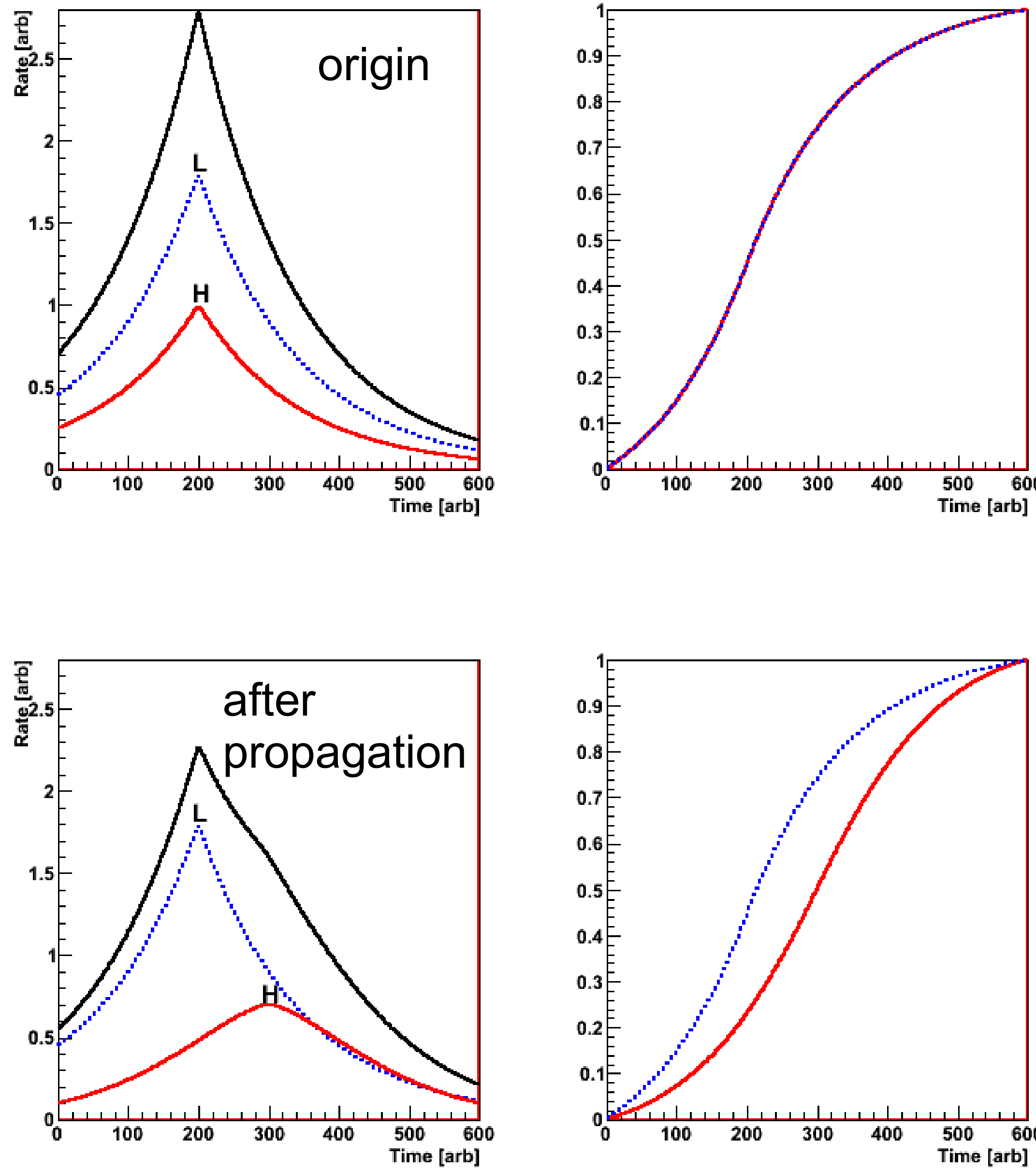
Methodology

At its simplest:

✗We do not know the intrinsic shape/width/duration of the light curve at origin.

✔We do know that for an energy dependent dispersion a skewness is introduced into the light curve shape.

Under the assumption of the high & low energy photons being emitted contemporaneously and co-spatially we can "de-disperse" the lightcurve until the low and high energy lightcurves match again.



Methodology

Apply an energy dependent correction factor τ to the event arrival time

$$\delta t_i = -\tau E_i^{\alpha}$$

where α defines the energy scale and
α=1 is linear dispersion
that is robust to energy resolution effects.

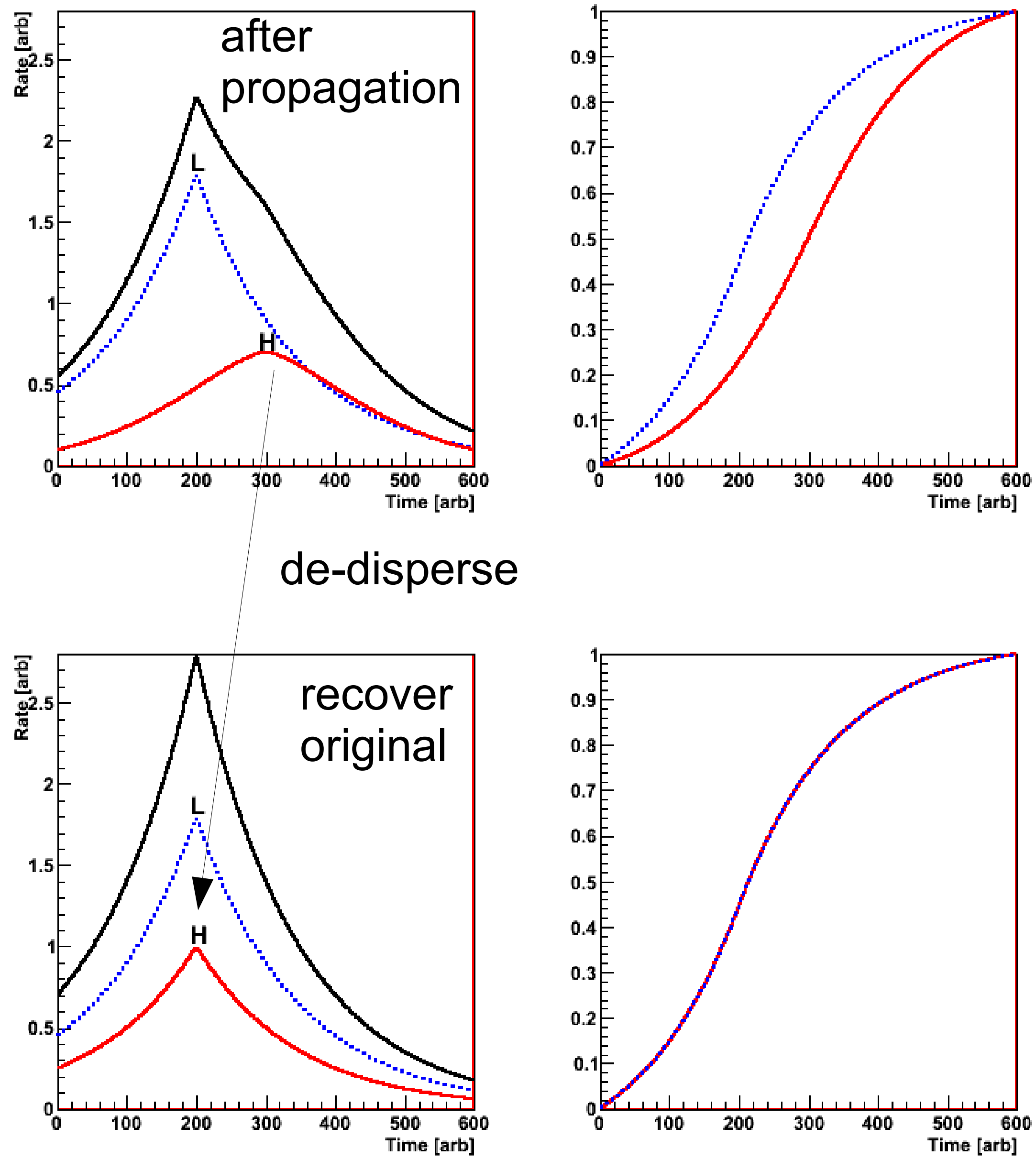
Minimise width:

e.g. DisCan & Shannon information
Scargle, Norris & Bonnell *ApJ* **673**, 972 (2008).

Maximise power:

e.g. Energy Cost Function on Mrk 501 light curve
MAGIC Collaboration *PhLB* **668**, 253 (2008).

but these still make an implicit assumption on the intrinsic light curve form being sharp.



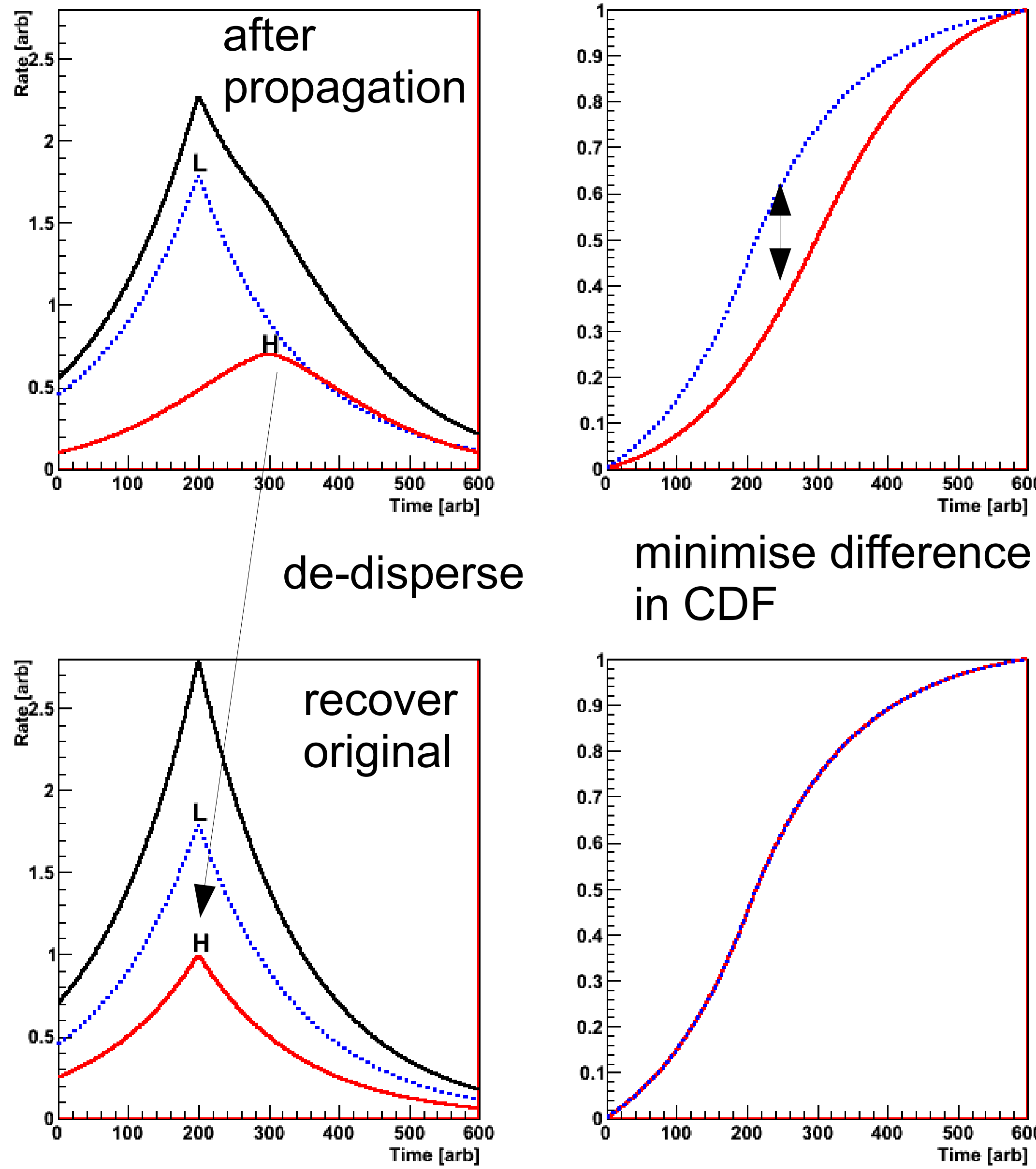
Apply an energy dependent correction factor τ to the event arrival time

$$\delta t_i = -\tau E_i^{\alpha}$$

where α defines the energy scale and α=1 is linear dispersion that is robust to energy resolution effects.

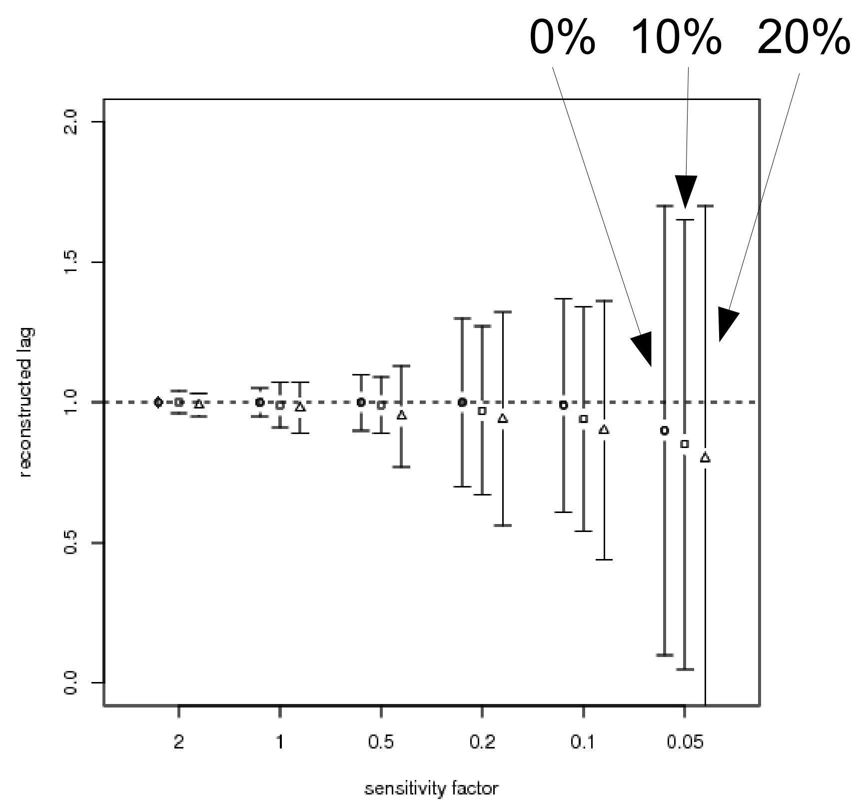
Here we merely assume that the low energy events can describe the form that the high energy lightcurve takes at source.

If the high energy events are emitted contemporaneously and co-spatially with the low energy ones then their cumulative distribution functions should overlap



Advantages of the Kolmogorov distance metric

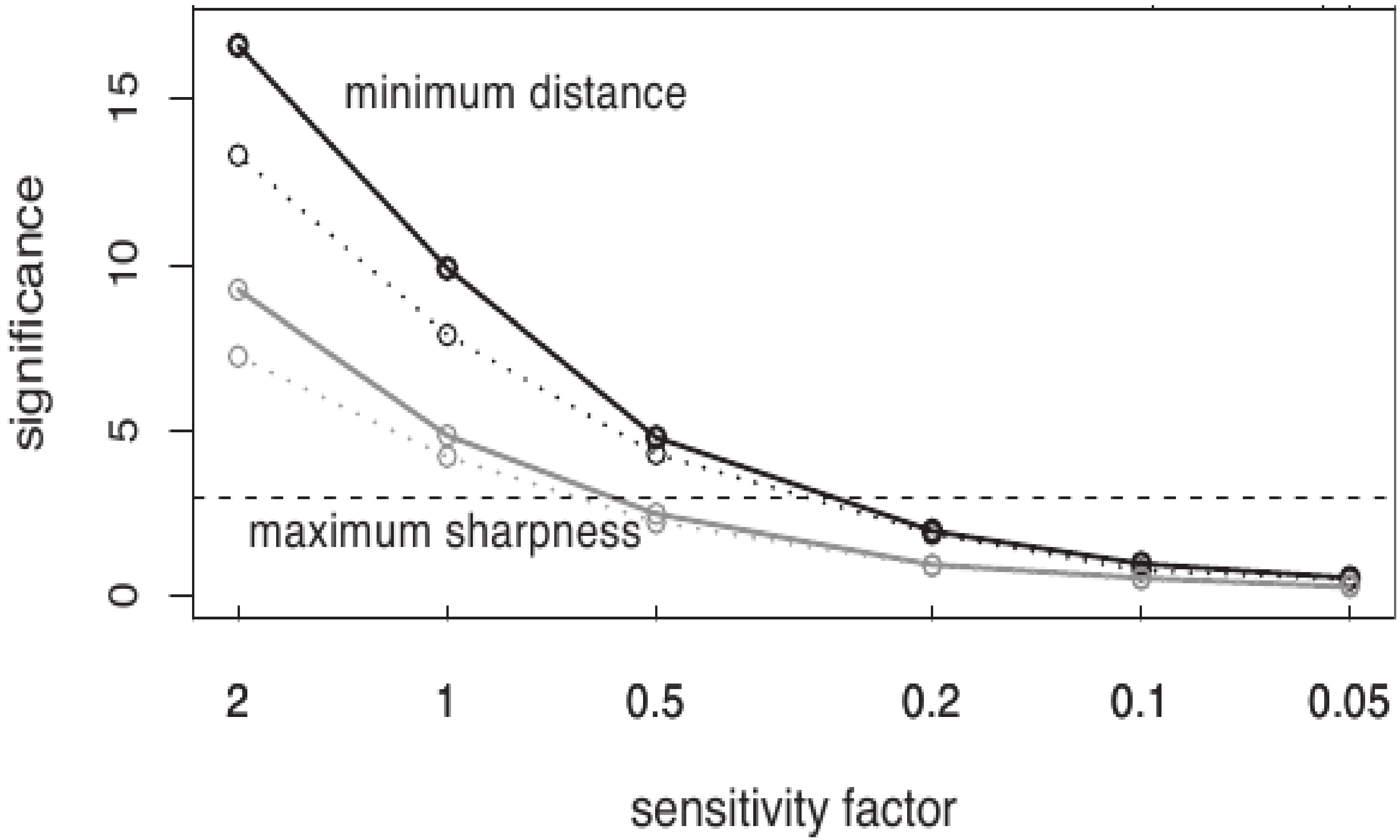
It is a fit to the entire profile, but lends a natural weight to the most transient portion of the profile
Averaging twice means it is less susceptible to statistical fluctuations and so can work with a small number of events
It is relatively insensitive to the energy resolution



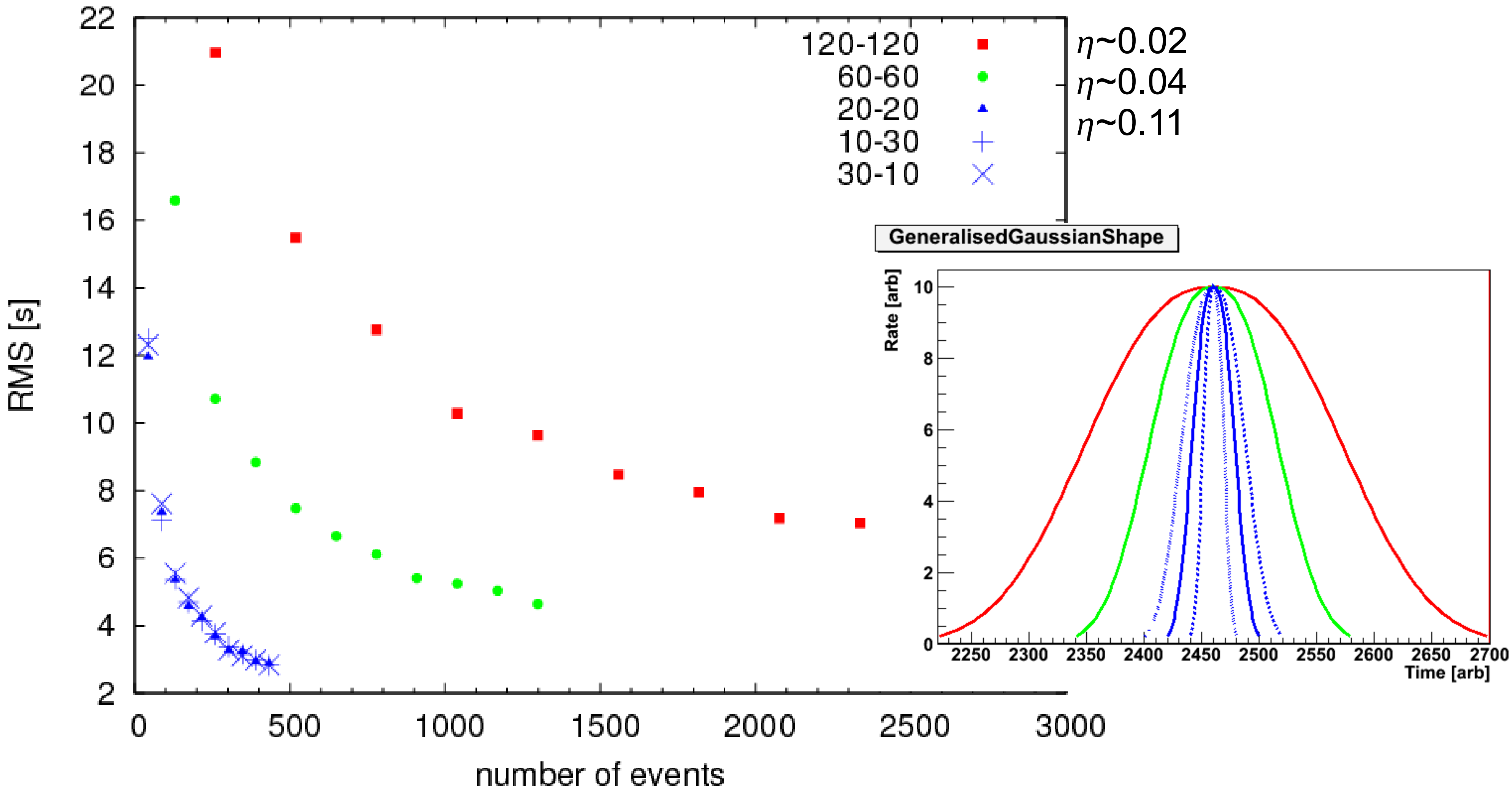
$$\eta = \frac{\delta t}{\Delta t}$$

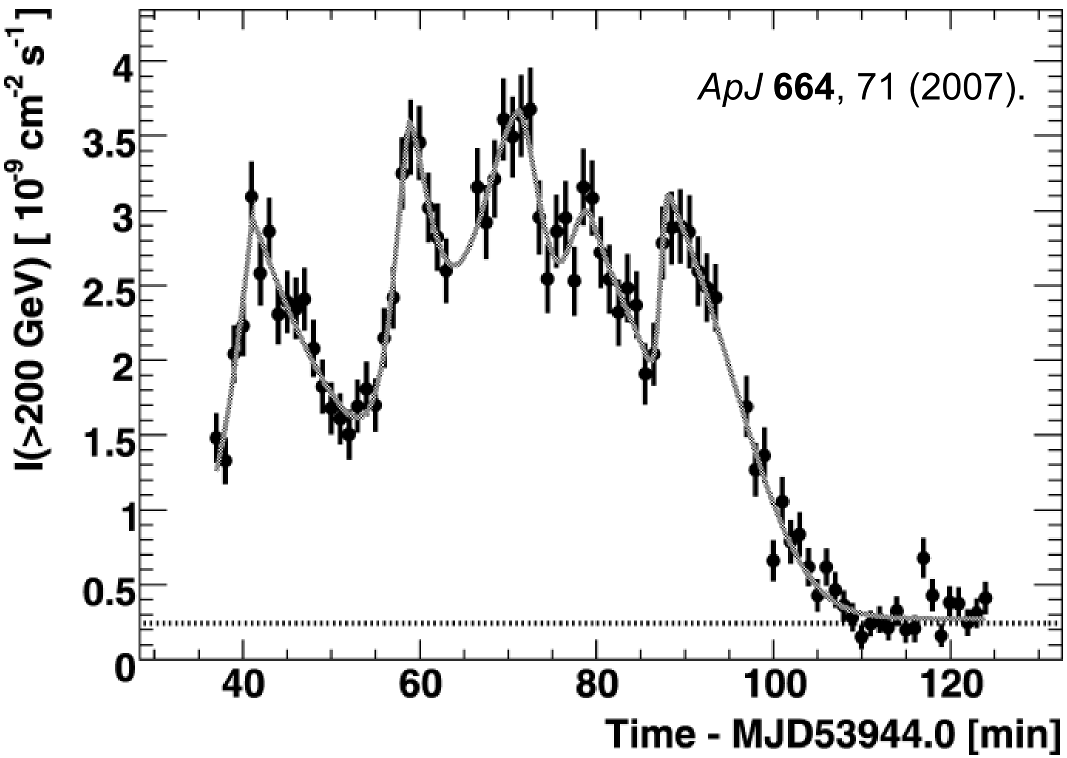
Relation to other cost functions

For PKS 2155-304
at z~0.113,
δt ~ 4.26 s per TeV
Δt $\geq$ 300s
η ~ 0.01



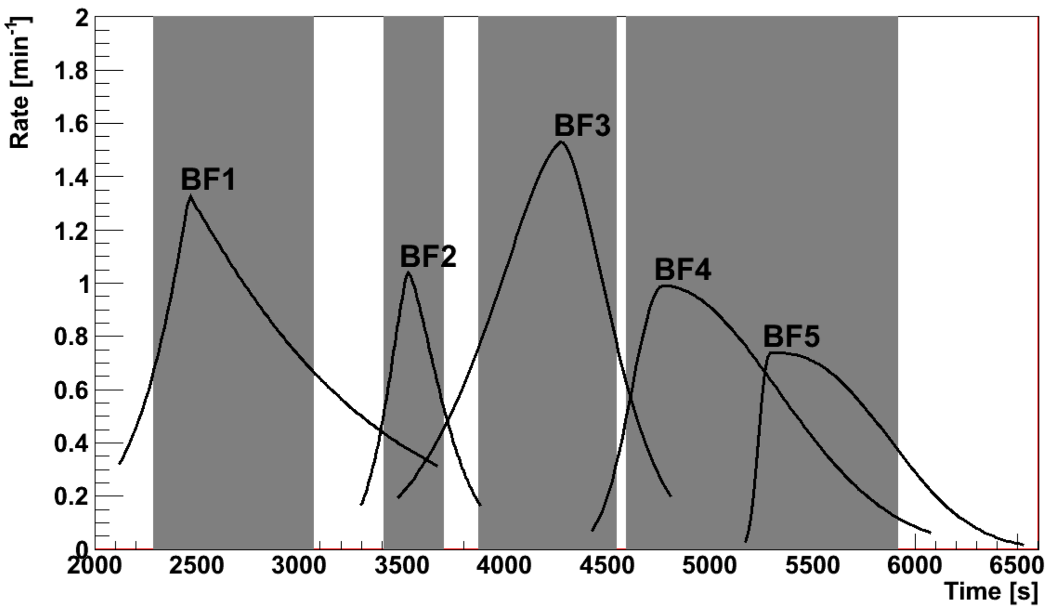
It is obviously going to be more sensitive to sharper flare/burst features, but it is definitely not sensitive to the light curve shape – performing equally well to symmetrical, fast/slow and slow fast rise/fall times.



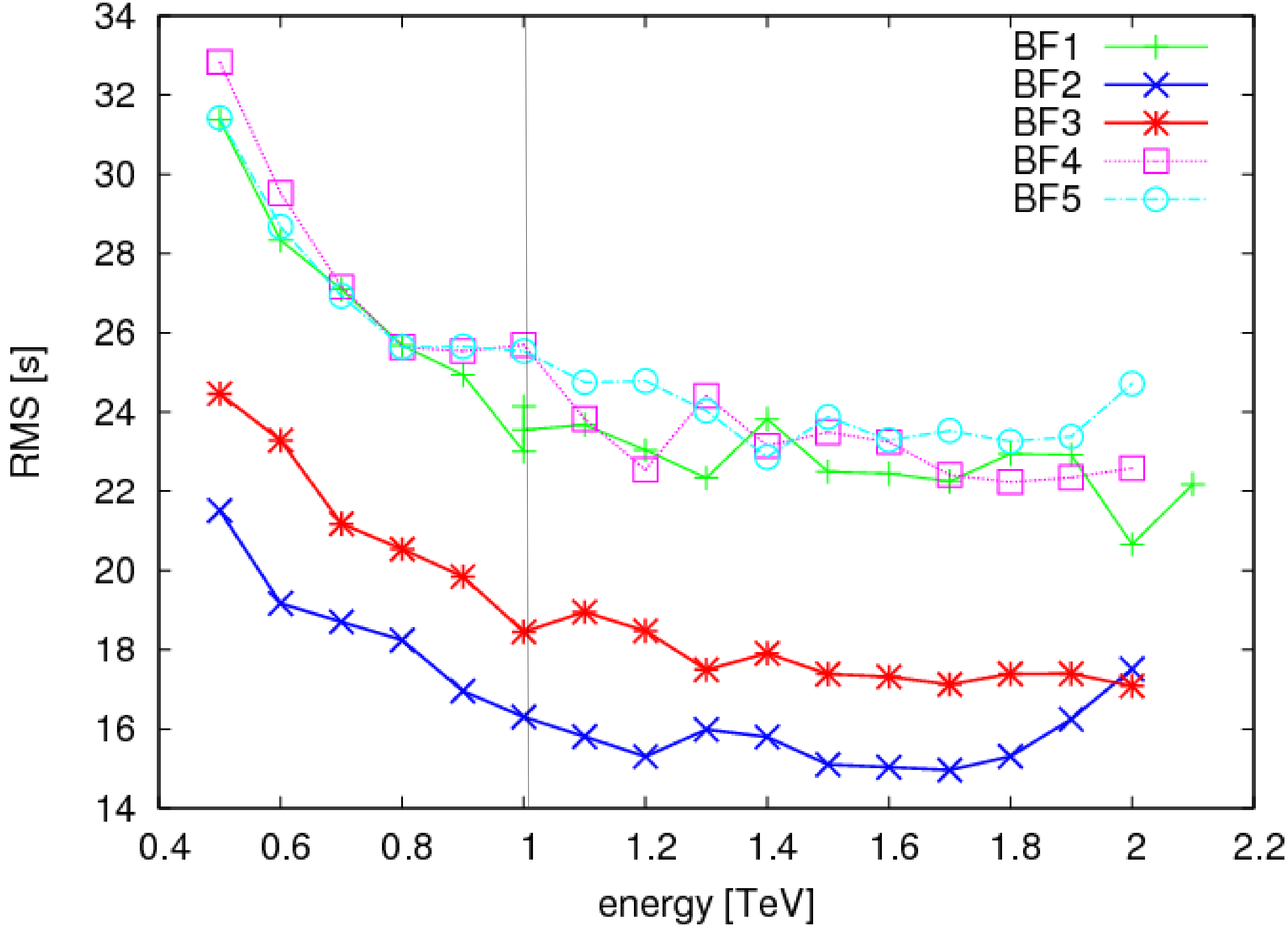


Flare	Window [s]	events < 0.5 TeV	events > E_{cut}	$< \Delta E >$ [TeV]
BF1	556	376	43	1.48
BF2	228	211	29	1.35
BF3	534	372	59	1.48
BF4	695	344	62	1.34
BF5	591	217	48	1.34

$$I(t) = I_{max} \exp\left(\frac{-|t - t_{max}|^{\kappa}}{\sigma_{r,d}} \right)$$

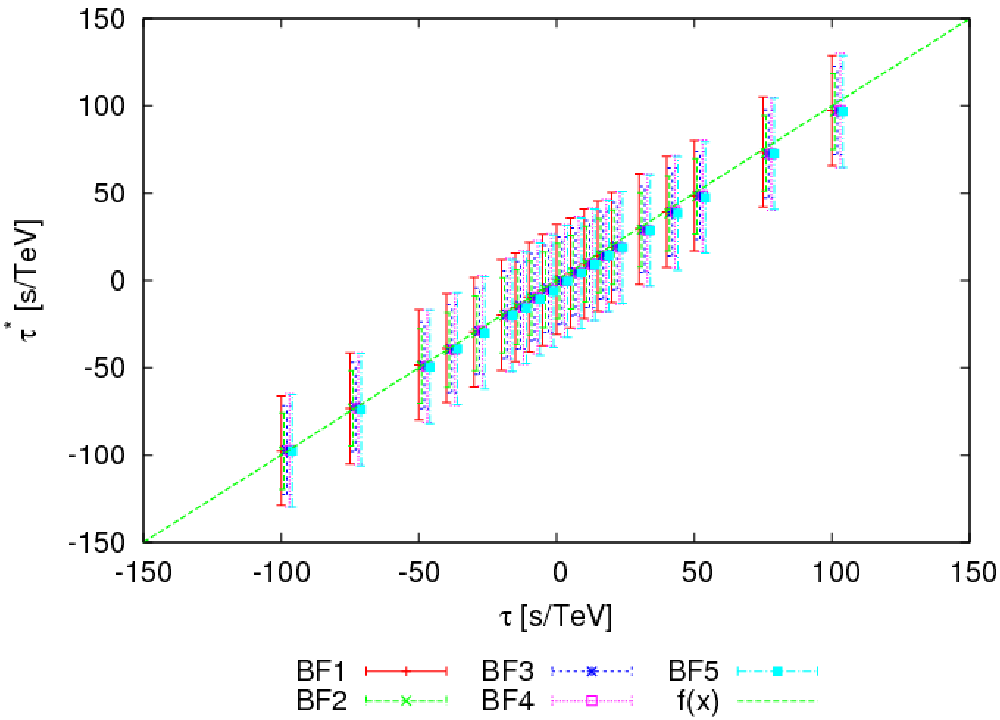


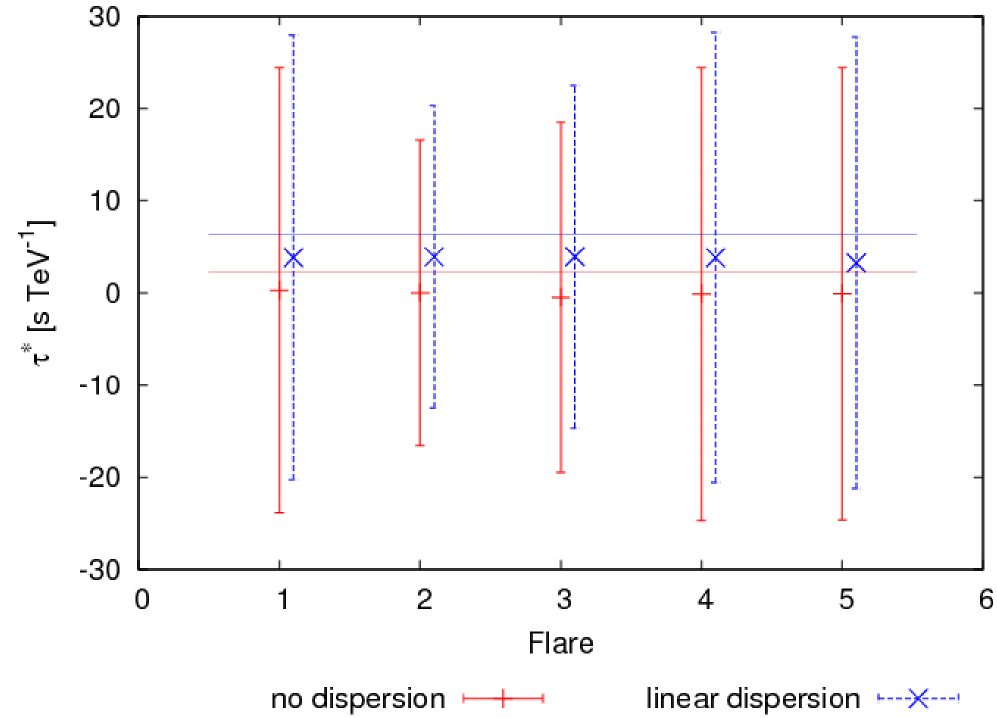
flare	t_{max} [s]	I_{max} [Hz]	σ_r [s]	σ_d [s]	κ
BF1	2460	1.33	173	610	1.07
BF2	3528	1.04	116	178	1.43
BF3	4278	1.53	404	269	1.59
BF4	4770	0.99	178	657	2.01
BF5	5298	0.74	67	620	2.44



It is relatively insensitive to the energy cuts until you start to run out of statistics

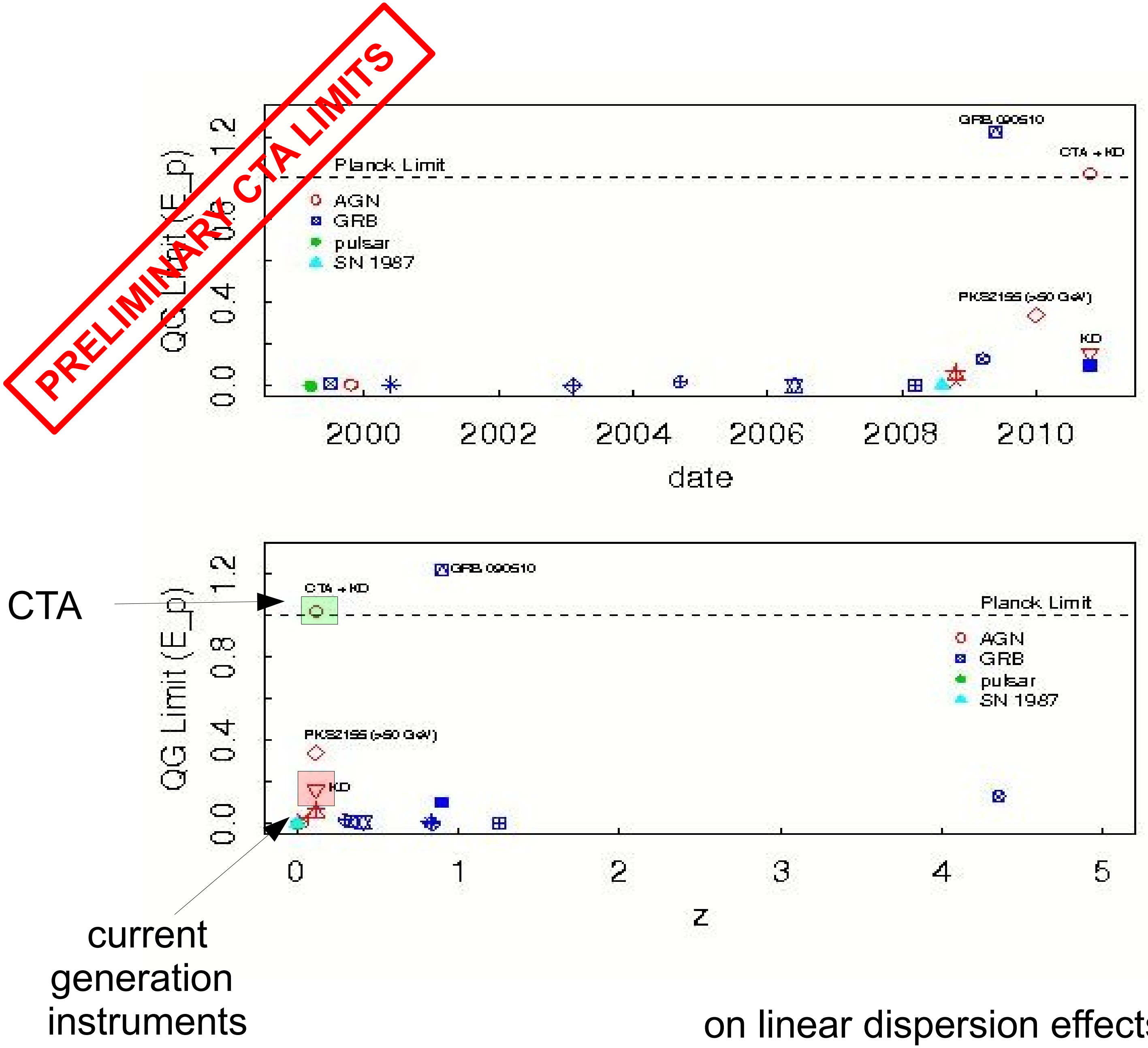
Recovering a Fixed Dispersion





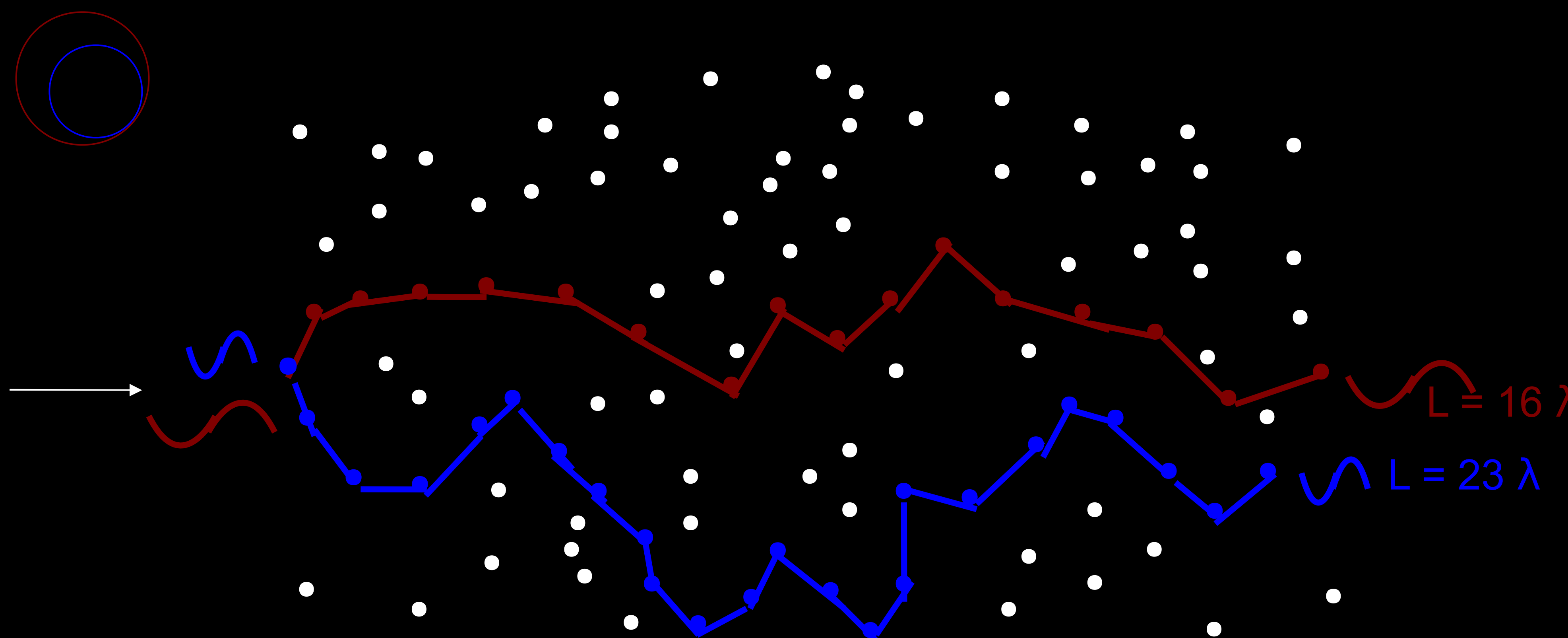
Whilst the RMS spread is larger than the expected dispersion for an individual flare when the burst width is so much greater than that dispersion, with a statistically large sample of flares the mean still appears to be an accurate estimate.

What can we expect from a big flare observed by CTA?



on linear dispersion effects...

Forward & Backward in time? Fuzzy dispersion: propagation through "foamy" space-time

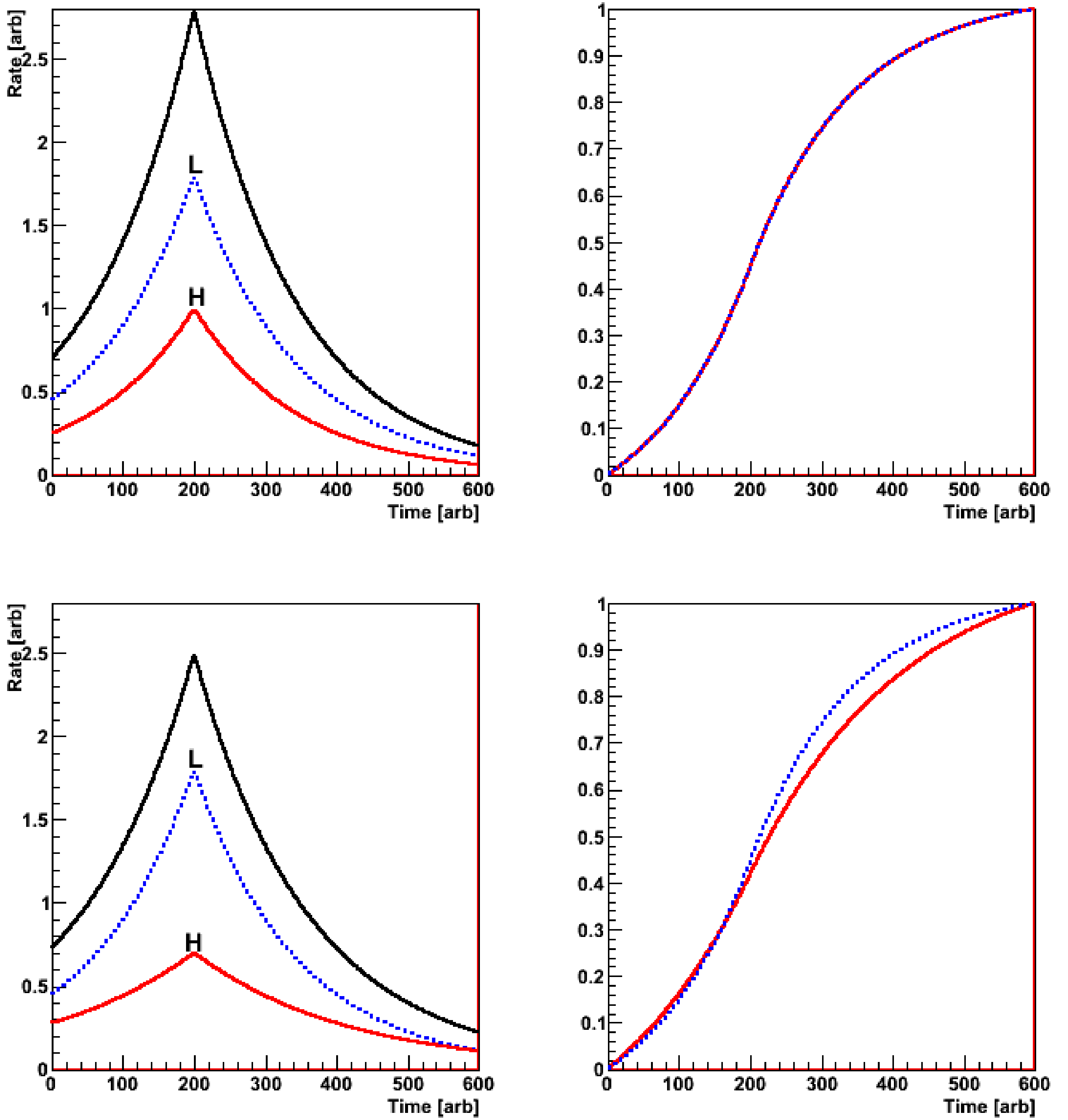


$$E \simeq p + \frac{m^2}{2p} - s_{\pm 1}\frac{1}{2}\frac{E^{\alpha+1}}{M_{QG}^{\alpha}}, \ \nu(E) \simeq 1 - \xi\frac{E}{M_{Pl}} \pm \xi\frac{E}{M_{Plk}\Delta\tau^{\text{¿}}} \pm \xi_f\frac{E}{M_{Pl}}$$

can be positive or negative

Amelino-Camelia & Smolin *PRD* **80**, 084017 (2009).

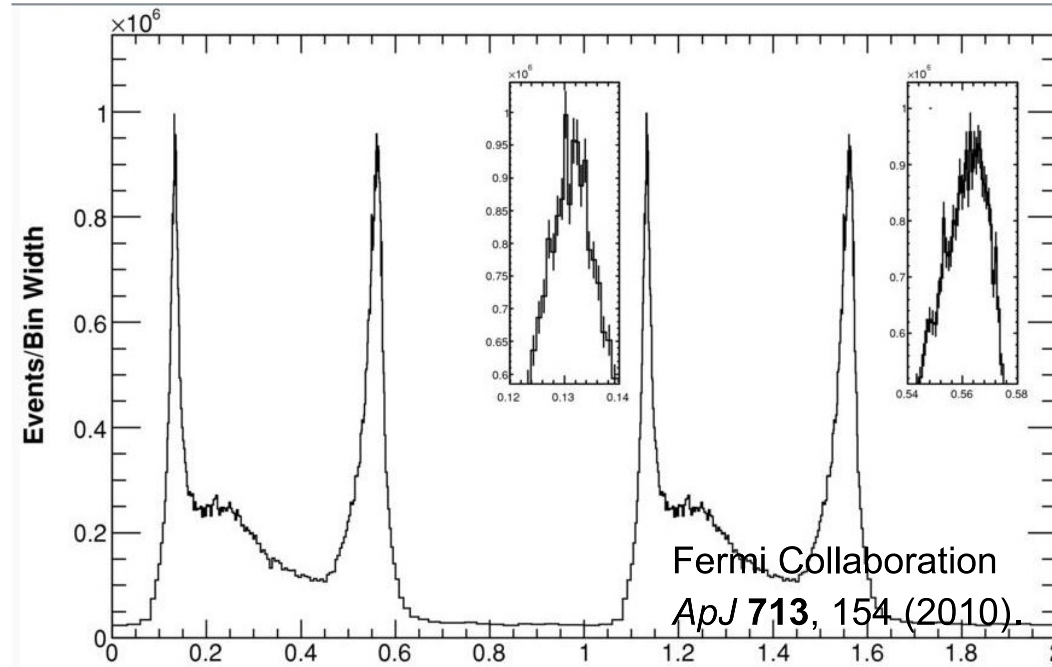
A negative co-efficient implies “superluminal” propagation can occur. If this happens as many times as sub-luminal then the lightcurve becomes broader, but the net displacement in the peak of emission is zero!



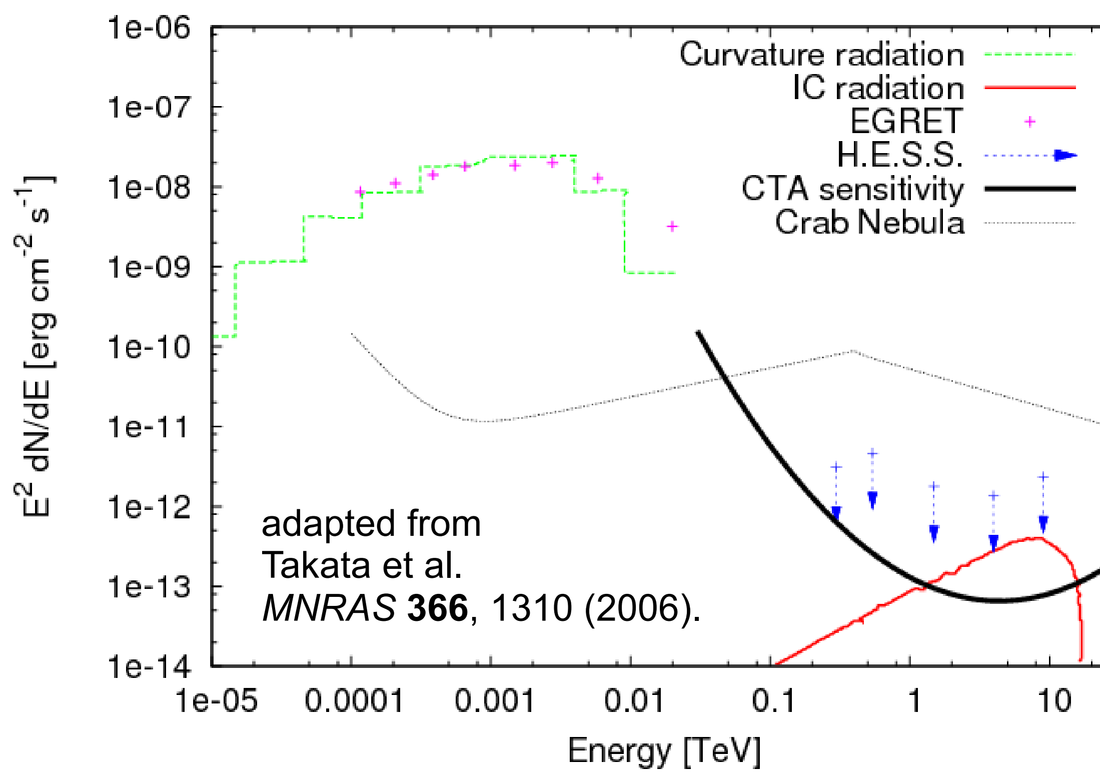
To test for superluminal effects we really need the sort of light curve that you know for sure where the maximum is. Could a pulsar lightcurve provide that reference point?

This is speculation,
but hopefully not an idle one

inverse Compton upscattered curvature radiation from pair creation across the outer gap could generate photons of a few TeV and possibly be detected by CTA



Vela pulsar



Summary

Unbinned tests are more sensitive when photon numbers are limited (e.g. short timescales)

A method that tried to minimise the number of assumptions about the light curve form has been presented

the prospects for CTA to be able to place Planck scale limits on (at least linear scale) Lorentz invariance violation effects are favourable.